

11-1-2017

## Basic Finance Made Accessible in Excel 2007: "The Big 5, Plus 2"

Stephen Sugden

*Bond University*, [ssugden@bond.edu.au](mailto:ssugden@bond.edu.au)

David Miller

*West Virginia University*, [millerd@math.wvu.edu](mailto:millerd@math.wvu.edu)

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### Recommended Citation

Sugden, Stephen and Miller, David (2017) Basic Finance Made Accessible in Excel 2007: "The Big 5, Plus 2", *Spreadsheets in Education (eJSiE)*: Vol. 4: Iss. 2, Article 1.

Available at: <http://epublications.bond.edu.au/ejsie/vol4/iss2/1>

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## Abstract

The basic principles and equations are developed for elementary finance, based on the concept of compound interest. The five quantities of interest in such problems are present value, future value, amount of periodic payment, number of periods and the rate of interest per period. We consider three distinct means of computing each of these five quantities in Excel 2007: (i) use of algebraic equations, (ii) by recursive schedule and the Goal Seek facility, and (iii) use of Excel's intrinsic financial functions. The paper is intended to be used as the basis for a lesson plan and contains many examples and solved problems. Comment is made regarding the relative difficulty of each approach, and a prominent theme is the systematic use of more than one method to increase student understanding and build confidence in the answer obtained. Full instructions to build each type of model are given and a complete set of examples and solutions may be downloaded (Examples.xlsx and Solutions.xlsx).

## Keywords

basic finance, spreadsheet, PV, FV, RATE, NPER, PMT, NPV, IRR

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# Basic Finance Made Accessible in Excel 2007: "The Big 5, Plus 2"

Steve Sugden  
Bond University, Australia  
ssugden@bond.edu.au

David Miller  
West Virginia University, USA  
millerd@math.wvu.edu

December 2010

## Abstract

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Full instructions to build each type of model are given and a complete set of examples and solutions may be downloaded ([Examples.xlsx](#) and [Solutions.xlsx](#)).

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Submitted September 2010, revised and accepted December 2010.

**Key words:** basic finance, spreadsheet, PV, FV, RATE, NPER, PMT, NPV, IRR.

## 1 Introduction

A sound understanding of basic finance is a very important and useful skill for life, even if a career in finance is not the ultimate goal. At least in western societies, everyone needs at least a basic understanding of the implications of borrowing or investing money in a compound interest environment. Traditional approaches to teaching this material to young people at school or university involved the use of algebra (geometric sequences and

their sums), and in many cases, merely plugging numbers into canned formulas, often with little understanding of the structure of the problem or with much inkling of what the answer might be. The use of calculators has, if anything, exacerbated this problem, with much mindless substitution of numbers into formulas and pushing buttons, then simply believing whatever emerges from the calculator or computer. With the advent of the modern electronic spreadsheet, as exemplified by Microsoft Excel, it is now not only possible, but rather easy to improve drastically on this approach to the teaching of basic finance.

In the spreadsheet environment, we advocate a three-pronged approach, and it is tacit that all three approaches must yield the same result for any given problem. The approaches are:

1. Traditional algebraic formulas, sometimes supplemented with the powerful *Goal Seek*
2. Linear recursive schedule, also sometimes with *Goal Seek*
3. Excel intrinsic financial functions

From the standpoint of student learning, it is important to note that the traditional algebraic approach has, in the majority of cases, and for several reasons, been less than adequate [9]. Put bluntly, many students have trouble with algebra. Further, an algebraic approach to even some elementary finance problems requires the use of logarithms (when the unknown is the number of periods), or even finding a zero of a high-degree polynomial (simple annuity when the unknown is the rate of interest). Student difficulties with logarithms are well-known [2], and, in our view, it is not reasonable to expect those learning elementary finance to be able to find roots of a polynomial of high degree. We submit, that with the spreadsheet-based approaches described in this paper, even students who struggle with elementary algebra are able to solve such problems and more with confidence, and a reasonable level of understanding. The second of our approaches listed above requires only the use of a linear recurrence. This is easily implemented in Excel, once students are comfortable with simple arithmetic expressions (formulas) in the spreadsheet environment. No logs or exponentials are required in this method; the intrinsic power of the recurrence relation is harnessed to produce the same effect. A side benefit is that the schedule thus produced can also be used to answer other finance questions, e.g., Lara just won the lottery and wishes to pay off her mortgage early, so what would the payout figure be? This amount can simply be read from the schedule with no further modelling or algebra whatever.

The approach just described has been used with success by the first author at both secondary and tertiary level, in Queensland, Australia. When constructing the spreadsheet models, we strongly advocate the liberal use of names rather than hard cell references. This facility, now significantly expanded in Excel 2007, allows our spreadsheet models to support cognitive links with the algebraic notation as written down on paper [1], [8]. In short, the Excel formulas look, as much as possible, like the algebraic ones.

In section 2 we review some fundamentals of elementary finance in a compound interest environment. In section 3 we derive the basic equation relating the fundamental five quantities: present value, payment, rate of interest, number of periods, and future value. We then consider how to solve this equation for each of the five quantities. Section 3.1 outlines our assumptions and derives the basic equation relating "the big five", while sections 3.2 through 3.6 go on to develop versions of this equation, targeted to solving for each of these five quantities. Section 4 gives detailed steps for using three different methods for solving for each of the five quantities in Excel. Finally, section 5 considers an extension of the techniques described earlier in the paper to solve the commonly-used *net present value* criterion for evaluation of projected cash flows. These techniques follow the same methods as introduced for earlier problems. The concept of *internal rate of return* is also covered in this final section. Some twelve exercises, with solutions, are provided at the end of the paper, and a full set of "live" Excel solutions is available for download. For further reading on the basics of compound interest and elementary finance, we recommend Knox *et al* [4] for the theory and examples, and MacDonald [5] for a good summary of Microsoft Excel's financial functions. The book [10] by Winston has treatment of some of the more advanced financial functions.

## 2 Basic principles of finance

### 2.1 Time value of money

It is a basic principle of finance that separate amounts of money cannot be equated, added or subtracted if they are at *different times*. Interest is present in all of the problems we consider. This means that, looking backward a dollar today is worth less yesterday, or, looking forward, it is worth more tomorrow. The principle of bringing all amounts to a common point, usually called a *pivot point*, is absolutely fundamental when solving finance problems. Complete freedom of choice is available for the pivot point, as the correct answer to a finance problem does not depend on the pivot point chosen. Therefore, the pivot point is usually chosen for convenience of calculation.

The fundamental mathematical topics on which elementary finance is based are the *arithmetic and geometric sequences*. Other terms in use are *AP* and *GP*, where the "P" stands for *progression*, an older term for sequence. Arithmetic sequences correspond to problems where simple interest is used, and geometric sequences to those involving compound interest. We do not consider *arithmetic sequences* and *simple interest* here.

### 2.2 Accumulation

Finding the value of an amount of cash at a point in the future is known as *accumulation*. The relevant formula is  $F = P(1 + r)^n$ , where  $P$  is the present value,  $F$  the future value,  $n$  the number of periods, and  $r$  the rate of compound interest per period. This reflects growth due to compound interest at a fixed rate.

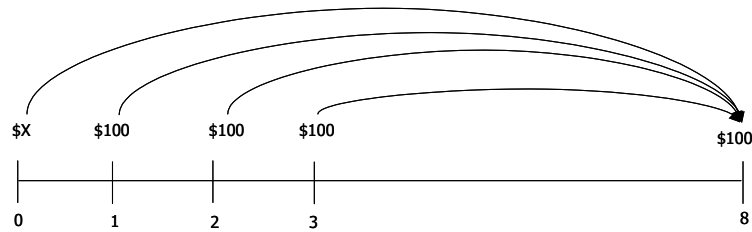


Figure 1: Basic timeline

### 2.3 Discounting

Finding the value of an amount of cash at a point in the past is known as *discounting*. The relevant formula is  $P = F(1 + r)^{-n}$ . This is, of course, equivalent to the formula for accumulation just given, except that we have solved for the present value,  $P$ . We may view this as "what amount,  $P$ , will accumulate to given amount,  $F$ , at rate  $r$  per period over  $n$  periods?".

### 2.4 Timelines

In the financial problems we consider, it will be found useful to use a kind of graph known as a *timeline*. This simple device gives us a reasonably accurate visual model of the problems and will be found to be a great aid to solving them. It is just a horizontal axis or number line, in which the variable is time, but annotated with amounts of money at specific times. It always starts at zero. The amounts are typically present or future values of loans or investments and regular payments. The aim is to then compare all monies at a common point, the so-called *pivot point*. The accumulation or discounting of cash is indicated by arrows on the timeline diagram. We write down an equation which balances all relevant amounts at the pivot point, and then seek to solve this equation for the desired unknown quantity.

For example, suppose eight quarterly payments of \$100 each exactly amortize a loan of \$ $X$ . A contract is signed where interest is accrued quarterly and the first payment is made one quarter from the date of the loan, shown as the point 0 on the timeline of Figure 1. Remembering the time value of money, to determine the amount of the loan, which we denote \$ $X$ , our method is to now accumulate all amounts to the point of the last payment, i.e.,  $q = 8$ , where  $q$  is the number of quarters elapsed from the original date of the loan. At that point, total debt must balance total payments. Such a problem is called a *present value annuity (PVA)*, since the unknown \$ $X$  is at zero. A timeline similar to that of Figure 1 might be used to illustrate the problem. We consider several problems of this nature, including some in the exercises at the end of the paper.



## 2.5 Example - term deposit

The simplest kind of problem involving compound interest is that of a term deposit. Here, the investor places a sum of money with a bank or similar institution, and simply leaves it there for an agreed period, and an agreed rate of interest. For example, at the birth of his daughter, a father decides to set up a term deposit which will be used to fund the girl's university education after she turns 18. To simplify the discussion, let us also suppose that university tuition and fees are projected to be \$120,000 when she is expected to attend university. If the fund earns 6% per annum, but compounded monthly, what amount must be invested to yield a maturity value of \$120,000? As outlined above, we examine three ways to solve this problem: algebra, recurrence in Excel, and finally, "black-box" formula in Excel.

## 3 The basic equation

We now give a general relationship between quantities involved in loans and investments. Compound interest is used. We first consider the "big five" Excel financial functions PV (present value), FV (future value), PMT (payment), RATE (rate of interest per period), and NPER (number of periods), then go on to look at two extensions: NPV (net present value) and IRR (internal rate of return).

### 3.1 Assumptions

We consider only the so-called *ordinary annuity*, in which:

1. Payments are made at the end of each period.
2. The frequency of payments matches that of compound interest updates.

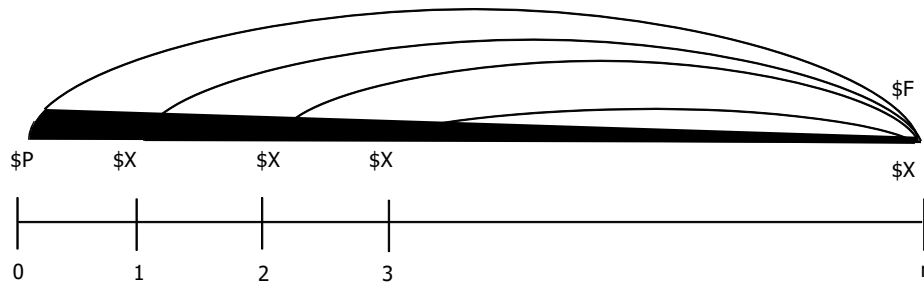


Figure 2: Timeline for ordinary annuity

We write  $P$  for the present value,  $F$  for the future value,  $X$  for the periodic payment,  $n$  for the number of periods, and  $r$  for the rate of compound interest per period.

The timeline of Figure 2 shows choice of the *pivot point*  $n$ . This means that all amounts not already there must be brought (i.e., accumulated or discounted) to that

point. Those going forward are *accumulated* while those going backward are *discounted*. Remember, to accumulate an amount at rate  $r$  per period for  $n$  periods, we multiply it by  $(1+r)^n$ . To discount an amount, we divide by the same quantity,  $(1+r)^n$ . In this case, everything is going forward (the amounts  $F$  and the final  $X$  are already there, so they need no adjustment at all). If we apply the principle that all monies must balance at any point in time, and choosing to "pivot" at  $n$ , we obtain the equation

$$F = P(1+r)^n + X + X(1+r)^1 + X(1+r)^2 + \dots + X(1+r)^{n-1} \tag{1}$$

Summing the geometric series in eq 1 yields

$$F = P(1+r)^n + X \left( \frac{(1+r)^n - 1}{r} \right) \tag{2}$$

If any four of the five quantities  $P, F, X, r, n$  are known, eq 2 may, in principle at least, be used to find the fifth. Except in the case where  $r$  is unknown, no great difficulty arises. In this case, the equation becomes rather difficult to solve algebraically, but an iterative method, such as that of *Newton* may be used effectively. We look at *Newton's method* in section 3.6.1.

In sections 3.2 through 3.6 we consider solving for each of the five variables in turn, leaving the difficult case of solving for  $r$  until last.

### 3.2 Solving for the future value, $F$

We may simply use eq 2 as is, i.e.,

$$F = P(1+r)^n + X \left( \frac{(1+r)^n - 1}{r} \right) \tag{3}$$

**Example 1 *Future Value Annuity: superannuation.*** Peter changes jobs a few months before his 45th birthday and receives a superannuation payout of \$445,000. He goes on a world cruise which costs him \$45,000, and he uses the remaining \$400,000 to start his superannuation fund with his new employer, who contributes \$800 per month to his retirement fund. If this fund earns 0.5% per month and Peter wishes to retire at age 60 (after exactly 15 years in the new job), what will be his lump sum payout at that time?

### 3.3 Solving for the present value, $P$

Beginning with eq 2, some algebra gives us

$$P = F(1+r)^{-n} - X \left( \frac{1 - (1+r)^{-n}}{r} \right) \tag{4}$$

**Example 2** At the birth of a son, a father uses a cash deposit \$ $P$  to open a fund, to be used for the son's tertiary education. He also pays \$200 per month into this fund, which earns  $\frac{1}{2}\%$  monthly. He estimates that Bond University fees will amount to \$120,000 when the son is 18 years old. How much must the initial cash deposit be?

### 3.4 Solving for the regular payment, $X$

Beginning with eq 2, some algebra gives us

$$X = \frac{r(F - P(1 + r)^n)}{(1 + r)^n - 1} \quad (5)$$

**Example 3 Present Value Annuity: a loan, mortgage or hire-purchase contract.** Theresa buys a used car, priced at \$13,500. She pays \$2250 down and is to pay the remainder in 36 equal monthly payments, the first due in 1 month. If the finance company charges 12% per annum, compounded monthly, how much is each payment?

**Remark 4** In the example just considered, the quantities on the right side of eq 5 take the values  $r = 1\% = 0.01$ ,  $F = 0$ ,  $P = 11250$ ,  $n = 36$ . In particular, we have  $F = 0$ . Eq 5 then reduces to

$$X = \frac{-rP(1 + r)^n}{(1 + r)^n - 1} \quad (6)$$

Since  $r > 0$  and  $P > 0$  it is clear from eq 6 that  $X < 0$ . This is consistent with the answers returned by the Excel intrinsic *PMT*, which we consider later.

### 3.5 Solving for the number of periods, $n$

We need to work slightly harder for this one. Clearing fractions in eq 2, we obtain

$$rF = rP(1 + r)^n + X(1 + r)^n - X \quad (7)$$

This, in turn, becomes

$$rF + X = (rP + X)(1 + r)^n$$

Then,

$$(1 + r)^n = \frac{rF + X}{rP + X}$$

Taking logarithms, we have

$$n \ln(1 + r) = \ln(rF + X) - \ln(rP + X)$$

Finally,

$$n = \frac{\ln(rF + X) - \ln(rP + X)}{\ln(1 + r)} \quad (8)$$

**Example 5** How many years will Steve have to wait to retire on \$2,000,000 if his superannuation fund began with \$200,000 rollover (*PV*), monthly contributions are \$500, and the fund earns 0.5% per month?

### 3.6 Solving for rate of interest, $r$

This is the tricky one. For our starting point, it is convenient to use eq 7. Considered as an equation in  $r$ , with the other quantities as parameters, this is a polynomial equation of degree  $n + 1$ . For further convenience, we write  $s = 1 + r$ . Equation 7 may be written

$$(s - 1)F = (s - 1)Ps^n + Xs^n - X$$

This equation can be rearranged to read

$$Ps^{n+1} + (X - P)s^n - Fs + (F - X) = 0 \tag{9}$$

We can usually solve this one for  $s$  by some computer-based method such as bisection, Newton's method, or even just using a graph and *Colour by Numbers* [6]. Then,  $r = s - 1$ . Note that it is quite easy to choose values of  $P, F, X, n$  so that a solution  $s > 1$  is impossible or nonsensical. For example, suppose  $P = 2000, F = 1000, X = 100, n = 10$ . We know that \$2000 cannot shrink to \$1000 over 10 periods while being augmented by regular payments of \$100, at least if the rate of interest is non-negative.

**Example 6** *An investor is seeking a relatively high-risk investment in order to get high returns. He is also prepared to inject a fixed amount of \$200,000 at the end of each year to boost the value of his investment. Under these conditions, what annual rate of growth would he need to find in order to make his initial investment of \$5,000,000 double in 5 years?*

#### 3.6.1 Newton's method

Newton's method is a (usually) rapidly-converging iterative technique to solve  $f(s) = 0$  for  $s$ . We choose a starting value,  $s_0$ . For the present application, and consistent with Excel's RATE function, we choose  $s_0 = 1.1$ , corresponding to  $r = 0.1 = 10\%$ . The iteration is:

$$s := s - \frac{f(s)}{f'(s)} \tag{10}$$

In this,  $f'(s)$  is the derivative of  $f(s)$  and we have used the Pascal-style update notation " := " (new value of  $s$  on the left and how to get it from the old value is on the right). For our application, we have, from eq 9:

$$\begin{aligned} f(s) &= Ps^{n+1} + (X - P)s^n - Fs + (F - X) \\ f'(s) &= (n + 1)Ps^n + n(X - P)s^{n-1} - F \end{aligned}$$

Substituting the expression for  $f(s)$  and  $f'(s)$  in eq 10, and after some simplification, we arrive at an expression for updating  $s$ , shown in eq 11.

$$s := \frac{nPs^{n+1} + (n - 1)(X - P)s^n - (F - X)}{(n + 1)Ps^n + n(X - P)s^{n-1} - F} \tag{11}$$

## 4 How to do all this in Excel

Excel has an enormous variety of inbuilt financial functions. We have promised to talk about the "big five". The five quantities are present value, future value, payment, rate and number of periods. A general relationship between these five was shown in equation 2. Given any four of the quantities, we may, at least in principle, solve for the remaining one. The algebraic approach to this has been discussed and it is clear that some problems are much easier to solve than others. Excel has a function corresponding to each of the "big five". These are **PV**, **FV**, **PMT**, **RATE**, **NPER**. For each one, the inputs are the remaining four; for example, the inputs to **PMT** are **PV**, **FV**, **RATE**, **NPER**. Each function also has another parameter, called **TYPE** which allows us to specify whether payments are made at the end or beginning of each period; in this paper, we always have payments at the end of each period<sup>1</sup>. Finally, the **RATE** function (rate of interest is the most difficult of the quantities to solve for) allows another parameter, being an initial guess for the rate value. This is required since direct algebra is not adequate to solve eq 9 and Excel employs an iterative method to solve, as discussed in section 3.6.1.

The following five sections (one corresponding to each of the problems of sections 3.2–3.6) give details of how to set up very simple Excel models to solve common problems in basic finance. For each of the problems, we consider the traditional algebraic approach (including an iterative method for the difficult case of *rate*), a linear recurrence technique (very simple but very powerful and transparent), and finally, use of Excel's intrinsic financial functions. The emphasis here will be placed on lucid modeling techniques, only invoking the intrinsic functions at the end in order to provide a check on the other two methods. Each section has three subsections, corresponding to the three methods of solution quantities of interest.

Before launching into the five problems, we briefly outline the three solution approaches in Excel.

### 4.1 Algebraic method using standard formulas or iteration

This is the traditional approach, and was used before the advent of electronic spreadsheets and financial/business calculators. We believe it is important to continue with this approach. However, in the interests of building student confidence in both the methods themselves and the answers obtained from them, we strongly recommend to supplement the algebraic approach with two other methods in Excel. These are the linear recurrence and intrinsic function methods, described in the sections to follow.

### 4.2 Linear recurrence (schedule) method

From a mathematical or information-theoretic point of view, recurrence relations pack a lot of information into equations which are often very simple-looking. For example, exponential relationships may be expressed in a linear fashion if we employ recurrences.

---

<sup>1</sup>Some finance problems, e.g., those involving payment of rents or insurance premiums often require payment to be made at the beginning of each period; we do not consider such problems here.

If we need to invert the relation (e.g., solve for number of periods rather than net worth) then logarithms are also implicitly involved. Since many students struggle with exponentials, and especially logarithms, this alone is reason enough for investigating the potential of linear recurrences to solve basic finance problems. Logs and exponentials may be avoided, and relationships may be expressed in the simplest of all possible mathematical models: the linear model. The recurrence approach also brings other benefits. As a natural by-product, it generates a schedule which in itself is very useful. It allows us to see the financial situation at any time. This may be the current or future net worth of a superannuation fund, or perhaps the total amount still owing on a loan. Armed with this information, it is a simple matter to answer questions such as "How much will my super fund be worth to me if I wish to retire five years earlier than planned?", or "I have just inherited some money and wish to pay off my mortgage early. How much must I pay?".

If we denote by  $w_m$  the worth of a fund at the end of month  $m$ , by  $r$  the rate of interest per month, and by  $X$  the monthly contribution, then the basic recurrence for FVA (superannuation) or PVA (loan or mortgage, with monthly payment  $\$X$ ) is  $w_m = (1 + r)w_{m-1} + X$ . For FVA,  $X$  is usually positive, and for PVA,  $X$  is always negative. The prototypical formula in Excel for  $m = 2$ , with  $w_m = w_2$  in E3 and  $w_{m-1} = w_1$  in E2 is `=E2 + rate*E2 + X`. This is the formula to be placed in E3 and then filled down to E182.

### 4.3 Intrinsic function method

Our final method is just to use Excel's inbuilt financial functions. This is generally simple enough, but there are some traps. Unless you are very experienced in using Excel's financial functions, we do not recommend this method by itself, but only as a check on the first two methods (algebraic formula and schedule). There are two principal reasons for this.

1. Excel makes assumptions about some of the parameters, which, at first sight, may appear to be somewhat peculiar. Payments and present values are expected to be negative. If you use the functions without knowing this, there is a very good chance you will get the wrong answer (in some instances, not even close to the right answer. For example, for the problem of section 3.3, use of the incorrect formula `=-PV(0.5%,216,200,120000)` yields \$67,240.85, whereas the true answer is \$14,481.70, obtained by use of the correct formula `=-PV(0.5%,216,-200,120000)`. Our advice is to play it safe and use at least one of the other two techniques described here, and unless you are very experienced, only use the inbuilt functions for verification. It is worth noting that a negative value for payment  $X$  is to be expected from eq 5 when we are dealing with a loan or mortgage. This is so, because usually  $F = 0$  (no residual payment) and  $P$  is often large and always positive. Assuming positive interest rates, the only way that eq 5 can be satisfied is for  $X$  to be negative. This is discussed further in sections 5 and 4.7.1.

- Using a built-in function is a totally opaque, black-box method, i.e., there is no insight into the structure of the problem. You cannot see how these functions operate except by experiment. It is akin to pressing a few buttons on a pocket calculator, and simply believing that whatever the machine displays is the correct answer. Unless you are very experienced indeed, this method, by itself, is fraught with peril.

#### 4.4 A word on local and global names in Excel 2007

As mentioned in the introduction, we are strong advocates of the disciplined use of names in spreadsheet modeling. This practice, where a cell or group of cells is given a name, which may then be used in formulas, generally leads to more understandable models. The quantities used in the spreadsheet may be given names directly related (or identical) to the corresponding algebraic quantities. Thus, for example, instead of the formula `=A12*(1+B16)^C12`, we prefer the wordy but more illustrative `=PresentValue*(1 + rate)^numperiods`. Excel 2007 has extended the power of names and contains a revamped "name manager". It is now possible to limit the scope (the particular worksheet of a workbook in which a name is visible and may be used in formulas). This means that names may be *global* to a workbook or *local* to a worksheet. The concept of local and global variables is well-known to computer programmers, but may be a brand-new concept for spreadsheet modelers. We adopt local names here, with some caution, given the likelihood that many readers may not be familiar with the idea of *name scope*. However, we believe that the benefits outweigh the risks, and we take care to carefully explain each instance of the process when it is required. Unfortunately, although the Excel 2007 names facility has been improved, the easiest ways to define names ("create from selection" and simply typing a name into the name box) only permit global scope of the newly defined names. This might be OK if the name manager then allowed us to change the scope of previously-defined name, but it does not. All of this forces us to use a more cumbersome method of defining our names, but the ability to take advantage of local variables (names) is too powerful to pass up. Details are given in the steps of model construction to follow.

#### 4.5 FV

We solve Example 1 of section 3.2 by each of the three methods.

- Open a fresh Excel workbook.

##### 4.5.1 Method 1: algebraic formula

- Rename the first sheet FV and select it.
- Place the headings `n`, `P`, `X`, `F`, `rate`, `(1 + rate)^n` in cells `A1:A6`.

3. Click in B1 and then Formulas tab, Defined Names panel, Define Name. Excel figures you want to define name  $n$ , based on the heading in A1. Just change the scope to FV (sheet) in the drop-down list. Close the dialog.
4. Do the same for P, X, F, rate.
5. Put the data 180, 400000, 800, 0.5% in cells B1,B2,B3,B5 respectively.
6. Format the cells B2:B4 as currency by selecting them and then clicking on the \$ button on the Number panel of the Home tab of the Ribbon.
7. We need the quantity  $(1 + r)^n$  twice in formula 3, so it makes sense to compute it once and make reference to whenever it is needed. This is a fundamental principle of spreadsheeting, and indeed, computing in general. So, put the formula  $=(1 + \text{rate})^n$  in B6.
8. Then, in B4, enter the formula  $=P*B6 + X*(B6-1)/\text{rate}$ . This is our rendering of formula 3 in Excel, given the names we have defined. A value of \$1,214,292.39 is found for the future value F.
9. Save your work.

#### 4.5.2 Method 2: linear recurrence

1. In cell D1 of the same worksheet just used for method 1, place the heading **month**. In E1 put **net worth at the end of the month**.
2. The relevant term (number of periods) for this problem is 180 (months). Place the integers 0 to 180 in the range D2:D182. This may be done in several ways; perhaps the simplest is to place the value 0 in D2, hold down the CTRL key, then drag the fill-handle of D2 down to D182.
3. Now the end of month 0 is the same as the beginning of month 1, so net worth there is P. Therefore, click in E2 then Formulas, Defined Names, Use in Formula, Select P. Excel will generate the formula  $=P$  for you. Since that is all we want, just hit Enter. Of course, you could just type  $=P$  in E2 for the same result. However, names are often more than one character and it is important to know that Excel will generate a reference (name) while constructing a formula if you click in the right place. Also, you won't misspell it if you pick from Excel's list of defined names.
4. Click on column E and format it as currency (click on \$ symbol, Number panel of the Home tab of the Ribbon).
5. This step is the heart of the model. Click in E3 and type the formula  $=E2 + E2*\text{rate} + X$ . Press Enter and the amount \$402,800.00 should appear.
6. Double-click E3's fill-handle, and *presto*, you have a schedule!



- The last entry in column E, viz., the contents of E182 represents the future value, F, of the investment and must agree with the answer obtained by the first method.
- Save your work.

#### 4.5.3 Method 3: intrinsic function

- In G1 of the same worksheet, type the formula  $=FV(\text{rate}, n, -X, -P)$ . If you prefer, you may instead use the Insert function dialog by clicking on  $f_x$  just to the left of the formula bar. When the Insert Function dialog appears, select the financial category, and then go to FV in the list box. Click OK, then fill-out the parameters for FV, being careful to include negations for X and P.
- Again, the contents of G2 represent the future value, F, of the investment and must agree with the answer obtained by the first two methods.
- Save your work.

| n            | 180             | month | net worth at the end of the month | Intrinsic      |
|--------------|-----------------|-------|-----------------------------------|----------------|
| P            | \$ 400,000.00   | 0     | \$ 400,000.00                     | \$1,214,292.39 |
| X            | \$ 800.00       | 1     | \$ 402,800.00                     |                |
| F            | \$ 1,214,292.39 | 2     | \$ 405,614.00                     |                |
| rate         | 0.50%           | 3     | \$ 408,442.07                     |                |
| (1 + rate)^n | 2.454093562     | 4     | \$ 411,284.28                     |                |
|              |                 | 5     | \$ 414,140.70                     |                |

Figure 3: Excel PV showing values

| n            | 180                           | month | net worth at the end of the month | Intrinsic         |
|--------------|-------------------------------|-------|-----------------------------------|-------------------|
| P            | 400000                        | 0     | =P                                | =FV(rate,n,-X,-P) |
| X            | 800                           | 1     | =E2 + rate * E2 + X               |                   |
| F            | =P * B6 + X * (B6 - 1) / rate | 2     | =E3 + rate * E3 + X               |                   |
| rate         | 0.005                         | 3     | =E4 + rate * E4 + X               |                   |
| (1 + rate)^n | =(1 + rate)^n                 | 4     | =E5 + rate * E5 + X               |                   |
|              |                               | 5     | =E6 + rate * E6 + X               |                   |

Figure 4: Excel PV showing formulas

## 4.6 PV

We solve Example 2 of 3.3 by each of the three methods.

#### 4.6.1 Method 1: algebraic formula

1. Rename the second sheet PV and select it.
2. Place the headings **n**, **P**, **X**, **F**, **rate**,  $(1 + \text{rate})^n$  in cells **A1:A6**. You can copy these from the previous sheet if you like.
3. Click in **B1** and then Formulas tab, Defined Names panel, Define Name. Excel figures you want to define name **n**, based on the heading in **A1**. Just change the scope to PV (sheet) in the drop-down list. Close the dialog.
4. Do the same for **P**, **X**, **F**, **rate**.
5. Put the data 216, 200, 120000, 0.5% in cells **B1**, **B3**, **B4**, **B5** respectively.
6. Format the cells **B2:B4** as currency by selecting them and then clicking on the \$ button on the Number panel of the Home tab of the Ribbon.
7. Put the formula  $=(1 + \text{rate})^n$  in **B6**.
8. Then, in **B2**, enter the formula  $=(F + X*(1 - B6)/\text{rate})/B6$ . This is our rendering of formula 4 in Excel, given the names we have defined. A value of \$14,481.70 is found for the present value P.
9. Save your work.

#### 4.6.2 Method 2: linear recurrence

1. In cell **D1** of the same worksheet just used for method 1, place the heading **month**. In **E1** put **net worth at the end of the month**.
2. The relevant term (number of periods) for this problem is 216 (months). Place the integers 0 to 216 in the range **D2:D218**.
3. We can pretend that we do not know the value of P obtained from the first method. Just put, say 20,000 in **E2**.
4. Click on column **E** and format it as currency (click on \$ symbol, Number panel of the Home tab of the Ribbon).
5. This step is the heart of the model. Click in **E3** and type the formula  $=E2 + E2*\text{rate} + X$ . Press Enter and the amount \$20,300.00 should appear.
6. Double-click **E3**'s fill-handle to get our schedule.
7. The last entry in column **E**, viz., the contents of **E218** represents the future value, **F**, of the investment and must agree with the value of **F** in cell **B4**. It does not, so we get Excel to alter our initial investment **P** until **218** matches **P** in cell **B4**.

8. Click in E218. Then go Data, Data tools, What-If Analysis, Goal Seek. Set cell E218 to value 120000 by changing cell E2. Click OK and Excel's Goal Seek tool will solve our problem. Examination of cell E2 will show the value \$14,481.70, in agreement with the value of P in B2 found in the first method.
9. Save your work.

#### 4.6.3 Method 3: intrinsic function

1. In G1 of the same worksheet, type the formula  $= -PV(\text{rate}, n, -X, F)$ .
2. If you prefer, you may use the Insert function dialog by clicking on  $f_x$  just to the left of the formula bar. When the Insert Function dialog appears, select the financial category, and then go to FV in the list box. Click OK, then fill-out the parameters for FV, being careful to include negations for X and P.
3. Again, the contents of G2 represent the present value, P, of the investment and must agree with the answer obtained by the first two methods.
4. Save your work.

| n                     | 216          | month | net worth at the end of the month | Intrinsic   |
|-----------------------|--------------|-------|-----------------------------------|-------------|
| P                     | \$ 14,481.70 | 0     | \$ 14,481.70                      | \$14,481.70 |
| X                     | \$ 200.00    | 1     | \$ 14,754.11                      |             |
| F                     | \$120,000.00 | 2     | \$ 15,027.88                      |             |
| rate                  | 0.50%        | 3     | \$ 15,303.02                      |             |
| $(1 + \text{rate})^n$ | 2.936765972  | 4     | \$ 15,579.53                      |             |
|                       |              | 5     | \$ 15,857.43                      |             |

Figure 5: Excel FV showing values

| n                     | 216                                | month | net worth at the end of the month | Intrinsic                      |
|-----------------------|------------------------------------|-------|-----------------------------------|--------------------------------|
| P                     | $=(F + X*(1 - B6)/\text{rate})/B6$ | 0     | 14481.6990946977                  | $= -PV(\text{rate}, n, -X, F)$ |
| X                     | 200                                | 1     | $=E2 + E2*\text{rate} + X$        |                                |
| F                     | 120000                             | 2     | $=E3 + E3*\text{rate} + X$        |                                |
| rate                  | 0.005                              | 3     | $=E4 + E4*\text{rate} + X$        |                                |
| $(1 + \text{rate})^n$ | $=(1 + \text{rate})^n$             | 4     | $=E5 + E5*\text{rate} + X$        |                                |
|                       |                                    | 5     | $=E6 + E6*\text{rate} + X$        |                                |

Figure 6: Excel FV showing formulas

## 4.7 PMT

We solve Example 3 of 3.4 by each of the three methods.

### 4.7.1 Method 1: algebraic formula

1. Rename the third sheet PMT and select it.
2. Place the headings **n**, **P**, **X**, **F**, **rate**,  $(1 + \text{rate})^n$  in cells A1:A6. You can copy these from the previous sheet if you like.
3. Define the names **n**, **P**, **X**, **F**, **rate** as in the previous examples, making sure they are local to the current worksheet.
4. Put the data 36, 11250, 0, 1% in cells B1, B2, B4, B5 respectively. Format B2:B4 as currency.
5. Put the formula  $= (1 + \text{rate})^n$  in B6.
6. Then, in B3, enter the formula  $= \text{rate} * (F - P * B6) / (B6 - 1)$ . This is our rendering of formula 5 in Excel, given the names we have defined. A value of  $-373.66$  is found for the payment **X**. Notice the negative value. The reason for this was discussed in section 3.4.
7. Save your work.

### 4.7.2 Method 2: linear recurrence

1. In cell D1 of the same worksheet just used for method 1, place the heading **month**. In E1 put **amount owing at the end of the month**.
2. The relevant term (number of periods) for this problem is 36 (months). Place the integers 0 to 36 in the range D2:D38.
3. In E2 put the formula  $=P$ . Format column E as currency.
4. This step is the heart of the model. Click in E3 and type the formula  $=E2 + E2 * \text{rate} + X$ . Press Enter and the amount \$10,988.84 should appear.
5. Double-click E3's fill-handle to get our schedule.
6. The last entry in column E, viz., the contents of E218 represents the future value, F, of the investment and should be zero. This is because it is using the correct payment value found from the previous method.
7. We now suppose that the correct payment value is unknown and use the schedule plus Goal Seek to find the payment. In F1 type the heading **payment**. Click in F2 and then go Formulas tab, Defined Names panel, Define Name. Ensure that your name is local to the current worksheet. Modify the formula in E3 to read  $=E2 + E2 * \text{rate} + \text{payment}$  and fill it down (double-click fill-handle of E3).

8. Click in E36. Then go Data, Data tools, What-If Analysis, Goal Seek. Set cell E36 to value 0 by changing cell payment. Click OK and Excel's Goal Seek tool will solve our problem. The value of payment as found in F2 will match the value  $-373.66$  already found.
9. Save your work.

#### 4.7.3 Method 3: intrinsic function

1. In H2 of the same worksheet, type the formula  $\text{=PMT}(\text{rate},n,P,0)$ .
2. The contents of H2 represent the amount of payment (negative for reasons already discussed) and must agree with the answer obtained by the first two methods.
3. Save your work.

| n                     | 36          | month | amount owing at the end of the month | payment     | Intrinsic   |
|-----------------------|-------------|-------|--------------------------------------|-------------|-------------|
| P                     | \$11,250.00 | 0     | \$ 11,250.00                         | $-\$373.66$ | $-\$373.66$ |
| X                     | $-\$373.66$ | 1     | \$ 10,988.84                         |             |             |
| F                     | \$0.00      | 2     | \$ 10,725.07                         |             |             |
| rate                  | 1%          | 3     | \$ 10,458.66                         |             |             |
| $(1 + \text{rate})^n$ | 1.430768784 | 4     | \$ 10,189.58                         |             |             |
|                       |             | 5     | \$ 9,917.82                          |             |             |

Figure 7: Excel PMT showing values

| n                     | 36                                 | month | amount owing at the end of the month                  | payment             | Intrinsic                        |
|-----------------------|------------------------------------|-------|---|---------------------|----------------------------------|
| P                     | 11250                              | 0     | =P  | $-373.660985394577$ | $\text{=PMT}(\text{rate},n,P,0)$ |
| X                     | $\text{=rate}*(F - P*B6)/(B6 - 1)$ | 1     | $\text{=E2} + \text{E2}*\text{rate} + \text{payment}$ |                     |                                  |
| F                     | 0                                  | 2     | $\text{=E3} + \text{E3}*\text{rate} + \text{payment}$ |                     |                                  |
| rate                  | 0.01                               | 3     | $\text{=E4} + \text{E4}*\text{rate} + \text{payment}$ |                     |                                  |
| $(1 + \text{rate})^n$ | $\text{=(1 + rate)}^n$             | 4     | $\text{=E5} + \text{E5}*\text{rate} + \text{payment}$ |                     |                                  |
|                       |                                    | 5     | $\text{=E6} + \text{E6}*\text{rate} + \text{payment}$ |                     |                                  |

Figure 8: Excel PMT showing formulas

## 4.8 NPER

We solve Example 5 of 3.5 by each of the three methods.

### 4.8.1 Method 1: algebraic formula

1. Rename the third sheet NPER and select it.

2. Place the headings **n**, **P**, **X**, **F**, **rate**,  $(1 + \text{rate})^n$  in cells **A1:A6**. You can copy these from the previous sheet if you like.
3. Define the names **n**, **P**, **X**, **F**, **rate** as in the previous examples, making sure they are local to the current worksheet.
4. Put the data 200000, 500, 2000000, 0.5% in cells **B2:B5** respectively. Format **B2:B4** as currency.
5. Then, in **B1**, enter the formula  $=\text{LN}(\text{rate}*\text{F} + \text{X}) - \text{LN}(\text{rate}*\text{P} + \text{X})/\text{LN}(1 + \text{rate})$ . This is our rendering of formula 8 in Excel, given the names we have defined. A value of 390.1541761 is found for the number of periods **n**. We interpret this to mean that 390 months is not quite enough to reach \$2M, but that 391 will take the total worth of the super fund to more than \$2M. Given that Steve is ageing rapidly (now in his late fifties), 390 months or 37.5 years, this is far too long to wait! Steve needs to drastically increase his contributions or find an investment with better returns (effective rate of interest).
6. Save your work.

#### 4.8.2 Method 2: linear recurrence

1. In cell **D1** of the same worksheet just used for method 1, place the heading **month**. In **E1** put **net worth at the end of the month**.
2. The relevant term (number of periods) for this problem is *a priori* unknown, although we have determined it to be 390 or 391 by the previous method. Pretending we don't know the term, place the values 0 through 400 in the range **D2:D402**.
3. In **E2** put the formula  $=\text{P}$ . Format column **E** as currency.
4. Click in **E3** and type the formula  $=\text{E2} + \text{E2}*\text{rate} + \text{X}$ . Press Enter and the amount \$201,500.00 should appear.
5. Double-click **E3**'s fill-handle to get our schedule.
6. Split the screen by going View, Window panel, Split (later, you can dismiss the vertical split by double-clicking on the vertical divider, if desired). Drag the horizontal split divider to about one-third way down the worksheet. Click somewhere in column **E** in the lower half of the split screen and type CTRL-down arrow. You should now see the last row of the schedule.
7. It should be clear by looking at months 390 and 391 that our schedule implies the same conclusion reached in the first method (algebraic formula).
8. If the schedule is long and you want to save your eyes, Excel's conditional formatting can help you. Click on column **E** and then go Home, Styles panel, Conditional formatting, Highlight cells rules, Greater than ...; then click in the text box on the

left of the dialog, and then click in the cell for F, i.e., in cell B4. Then click OK. CF will do its thing and you will easily see the answer highlighted in the schedule in column E.

9. Save your work.

#### 4.8.3 Method 3: intrinsic function

1. In G2 of the same worksheet, type the formula `=NPER(rate,-X,-P,F)`. Note the negations required for X and P.
2. The value 390.1542 appears in G2 (may be slightly different due to formatting. You can double-click on the divider between columns G and H to get more accuracy: 390.1541761.
3. Save your work.

| n    | 390.1541761     | month | net worth at the end of the month | Intrinsic   |
|------|-----------------|-------|-----------------------------------|-------------|
| P    | \$ 200,000.00   | 0     | \$ 200,000.00                     | 390.1541761 |
| X    | \$ 500.00       | 1     | \$ 201,500.00                     |             |
| F    | \$ 2,000,000.00 | 2     | \$ 203,007.50                     |             |
| rate | 0.50%           | 3     | \$ 204,522.54                     |             |
|      |                 | 4     | \$ 206,045.15                     |             |

Figure 9: Excel NPER showing values

| n    | $=(\text{LN}(\text{rate}*\text{F} + \text{X}) - \text{LN}(\text{rate}*\text{P} + \text{X}))/\text{LN}(1 + \text{rate})$ | month | net worth at the end of the month | Intrinsic           |
|------|---|-------|-----------------------------------|---------------------|
| P    | 200000  | 0     | =P                                | =NPER(rate,-X,-P,F) |
| X    | 500   | 1     | =E2 + E2*rate + X                 |                     |
| F    | 2000000   | 2     | =E3 + E3*rate + X                 |                     |
| rate | 0.005   | 3     | =E4 + E4*rate + X                 |                     |
|      |   | 4     | =E5 + E5*rate + X                 |                     |

Figure 10: Exce INPER showing formulas

## 4.9 RATE

We solve Example 6 of 3.6 by each of the three methods.

### 4.9.1 Method 1: algebraic formula

1. Rename the third sheet RATE and select it.

2. Place the headings **n**, **P**, **X**, **F**, **rate**, **s<sup>n</sup>**, **s**, **leftside** in cells **A1:A8**.
3. Define the names **n**, **P**, **X**, **F**, **rate** as in the previous examples, making sure they are local to the current worksheet. Also define the name **s** to refer to cell **B7**.
4. Put the data 5, 5000000, 200000, 10000000 in cells **B1:B4** respectively. Format **B2:B4** as currency.
5. Put the formula `=s - 1` in **B5**.
6. Put the formula `=s^n` in **B6**.
7. Put the formula `=P*B6*s + (X - P)*B6 - F*s + F - X` in **B8**. This represents the left side of eq 9.
8. Eq 9 is a polynomial equation in  $s$ . It cannot usually be directly solved by algebra. However, we could use a powerful method such as Newton's method in Excel. This requires an initial estimate of  $s$ , which we take as  $s_0 = 1.1$ . As noted in section 3.6.1, this is consistent with the default initial guess of 10% in Excel's RATE function. We do not implement Newton's method here, but leave that as an exercise for the reader. Instead, we employ Goal Seek.
9. In **B7**, place the initial value 1.1 for  $s$ .
10. Click again in **B8**. Go Data, Data tools, What-If Analysis, Goal Seek. Set cell **B8** to value 0 by changing cell **s**. Click OK and Excel's Goal Seek tool will solve our problem. With default formatting for cells **B5**, the value appearing in this cell after Goal Seek is 0.118027836. If we want this as a percentage, just click on the % formatting button on the Number panel of the home tab of the Ribbon with **B5** selected. The value 12% appears. Since more accuracy is required, keep hitting the Increase Decimal button in the same panel until you have 7 decimal places. The value should finally read 11.8027836%.
11. Save your work.

#### 4.9.2 Method 2: linear recurrence

1. In cell **D1** of the same worksheet just used for method 1, place the heading **year**. In **E1** put **net worth at the end of the year**.
2. The relevant term (number of periods) for this problem is 5 (years). Place the integers 0 to 5 in the range **D2:D7**.
3. In **E2** put the formula `=P`. Format column **E** as currency.
4. Click in **E3** and type the formula `=E2 + E2*rate + X`. Press Enter and the amount \$5,790,139.18 should appear.
5. Double-click **E3**'s fill-handle to get our schedule.



6. It should be clear by looking at E7 that the rate in B5 is generating the \$10M as required in E7.
7. If we now suppose that the rate is not known, Goal Seek could also be used to get the correct value in E7. This is left as an exercise for the reader.
8. Save your work.

#### 4.9.3 Method 3: intrinsic function

1. In G2 of the same worksheet, type the formula `=RATE(n,-X,-P,F,,10%)`. Note the negations required for X and P, and the defaulted fifth parameter, as indicated by two consecutive commas.
2. The value 12% appears in G2. Use the format paintbrush to copy the formatting of B5 to G2.
3. Save your work.

| n        | 5                | year | net worth at the end of the year | Intrinsic   |
|----------|------------------|------|----------------------------------|-------------|
| P        | \$ 5,000,000.00  | 0    | \$ 5,000,000.00                  | 11.8027836% |
| X        | \$ 200,000.00    | 1    | \$ 5,790,139.18                  |             |
| F        | \$ 10,000,000.00 | 2    | \$ 6,673,536.77                  |             |
| rate     | 11.8027836%      | 3    | \$ 7,661,199.87                  |             |
| s^n      | 1.746880036      | 4    | \$ 8,765,434.71                  |             |
| s        | 1.11803          | 5    | \$ 10,000,000.00                 |             |
| leftside | -2.75671E-07     |      |                                  |             |

Figure 11: Excel RATE showing values

| n        | 5                                  | year | net worth at the end of the year | Intrinsic             |
|----------|------------------------------------|------|----------------------------------|-----------------------|
| P        | 5000000                            | 0    | =P                               | =RATE(n,-X,-P,F,,10%) |
| X        | 200000                             | 1    | =E2 + E2*rate + X                |                       |
| F        | 10000000                           | 2    | =E3 + E3*rate + X                |                       |
| rate     | =s - 1                             | 3    | =E4 + E4*rate + X                |                       |
| s^n      | =s^n                               | 4    | =E5 + E5*rate + X                |                       |
| s        | 1.11802783558387                   | 5    | =E6 + E6*rate + X                |                       |
| leftside | =P*B6*s + (X - P)*B6 - F*s + F - X |      |                                  |                       |

Figure 12: Excel RATE showing formulas

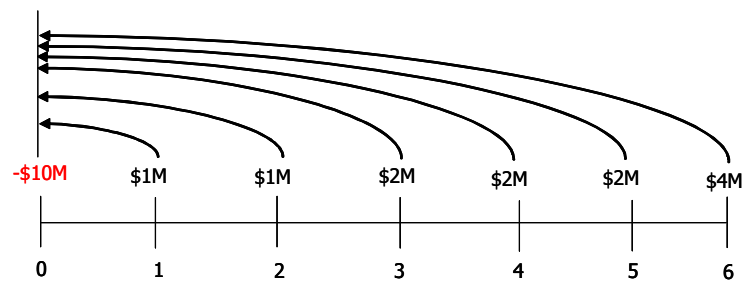


Figure 13: Timeline for NPV problem

## 5 Net present value

We have considered the five fundamental quantities in basic finance, their interrelationships, and how to solve a range of elementary finance problems in Excel. We now go on to consider some extensions to these ideas.

For many large businesses, a frequently-used tool to help decide whether to embark on a proposed new venture is *Net Present Value* or NPV. It is based on the concept of *discounted cash flows*. There is always the cost associated with setting up of the venture, and there may often be further costs along the way. Of course, there must be some points where income is received! The overall balance of cash flows determines whether the venture should proceed. We set up a timeline, as usual, to model our known or projected cash flows—some may be guesses, but we do the best we can!

Here is an example. Suppose our initial set-up cost is \$10M and we expect to receive amounts \$1M, \$1M, \$2M, \$2M, \$2M, \$4M at the end of each year thereafter. We illustrate this on a timeline, shown in Figure 13. This information may also be expressed in a table in Excel as shown in Figure 14. Simple addition of these amounts gives us a net inflow (profit) of \$2M. However, things are not so simple. We have already learned that amounts of cash at different times cannot just be added. Remember "time value of money"? Before such amounts can be compared, added or subtracted, they must all be brought (i.e., accumulated or discounted) to a common point in time. Since we are working in the present ("now"), and trying to get an estimate of the net flow, we choose to discount them all back them all to 0 on our timeline. Hence the name, *Net Present Value*. The method used is just as before, as we use our familiar discounting formula  $P = F(1 + r)^{-n}$ . This is just eq 4 with  $X = 0$ .

Discounted cash flows are shown in Figure 14, and the relevant formulas in Figure 15.

### 5.1 Some comments on NPV

1. The proposed venture is considered worthwhile if the NPV is positive, and not worthwhile if NPV is negative.

| Year                  | Cash flow at end year | PV of cash flow | rate |
|-----------------------|-----------------------|-----------------|------|
| 0                     | -\$10,000,000         | -\$10,000,000   | 1.0% |
| 1                     | \$1,000,000           | \$990,099       | 10   |
| 2                     | \$1,000,000           | \$980,296       |      |
| 3                     | \$2,000,000           | \$1,941,180     |      |
| 4                     | \$2,000,000           | \$1,921,961     |      |
| 5                     | \$2,000,000           | \$1,902,931     |      |
| 6                     | \$4,000,000           | \$3,768,181     |      |
| Using schedule, NPV = |                       | \$1,504,648     |      |

Figure 14: Excel NPV model showing values

| Year   | Cash flow at end year | PV of cash flow     | rate     |
|--|-----------------------|---------------------|----------|
| 0  | -10000000             | =B2/(1 + rate)^year | =D3/1000 |
| 1  | 1000000               | =B3/(1 + rate)^year |          |
| 2  | 1000000               | =B4/(1 + rate)^year |          |
| 3  | 2000000               | =B5/(1 + rate)^year |          |
| 4  | 2000000               | =B6/(1 + rate)^year |          |
| 5  | 2000000               | =B7/(1 + rate)^year |          |
| 6  | 4000000               | =B8/(1 + rate)^year |          |
| Using schedule, NPV =                                  |                       | =SUM(C2:C8)         |          |
| Using inbuilt function, NPV of all but first cash flow |                       | =NPV(rate,B3:B8)    |          |
| Final NPV after adding in first cash flow              |                       | =C13+B2             |          |

Figure 15: Excel NPV model showing formulas

2. The value of NPV depends on the cash flows as well as the rate of interest. NPV is usually positive for some interest rates and negative for others.
3. As we vary the rate of compound interest, there is usually a kind of "break-even" point where NPV is zero. This rate is known as the *internal rate of return*. In Excel, the function is IRR. Some cash flows do not have an internal rate of return, while others have more than one. In either situation, we abandon the entire IRR concept as it becomes meaningless. We discuss IRR further in section 5.3.

## 5.2 NPV Example

An investment over a term of ten years is proposed. It requires an initial outlay of \$100M, and an additional expenditure of \$25M after five years. Projected cashflows (all positive, i.e., incoming revenue) at the end of years 1 to 10 are respectively (all in \$M) 5, 8, 10, 6, 15, 22, 17, 28, 33, 45. Construct an Excel model which finds NPV of this series of cashflows from first principles. Then use the Excel function NPV to verify your answer. In your model, make use of a slider to control the annual rate of interest with increments of 0.001%. Once chosen by setting the slider, the rate is constant for all cash flows.

## 5.3 IRR

We mentioned above the concept of *internal rate of return* (IRR) of a series of cash flows. It is the rate of interest in an NPV model such that we break even. In other words, what rate in an NPV model would give a net present value of zero? Excel has a built-in function IRR for this purpose. We may also use our NPV model and invoke Goal Seek to find the rate giving an NPV of zero. If we do this for the example of section 5, the internal rate of return turns out to be approximately 4.446%.

Try it out for the NPV example of section 5.2. It must be kept in mind that sometimes IRR does not exist. A rule that guarantees that IRR will exist is that the model's cash flows have exactly one change in sign. Most capital investment projects begin with a negative cash flow (representing initial costs), followed by a sequence of (projected) positive cash flows. In this case, there is guaranteed to be a unique IRR.

## 6 Conclusion

We have presented a unified approach to basic finance in the modern spreadsheet environment. It is a three-pronged approach which emphasizes understanding of model structure over use of "black-box" functions. The "big five" quantities: present value, payment, rate of interest, number of periods, and future value are related via a fundamental equation. With minimal algebra, it is shown how to solve for each of the quantities and to develop a high level of confidence in the answer obtained. This confidence is built via use of alternative methods for checking correctness of solutions. Detailed steps are given for the construction of spreadsheet models to solve a wide variety of problems. The power

of the recurrence is exploited in the spreadsheet environment, and students may operate with simple linear relationships only to solve their problems. Once confidence is built, alternative methods which require knowledge of exponential and logarithmic functions may be introduced. Finally, Excel's intrinsic functions are used to compare and verify answer obtained by the first two approaches. We claim that a greater depth of understanding may be reached, and with less effort than by using other technologies such as graphics calculators or financial/business calculators. Finally, the modern spreadsheet is a ubiquitous tool in daily life. Students who develop a practical working knowledge of this tool while at school or university then have a skill greatly valued in the business world: the ability to work competently with electronic spreadsheets.

Mathematical relationships expressed as recurrences are often much simpler in form than otherwise equivalent relationships expressed "directly". We have considered the classic example of ordinary annuities, where exponential functions may be avoided by working with equivalent linear recurrences. Since this approach requires a schedule to be constructed, there is no reason why, using this basic framework, more general annuities may not also be studied in the spreadsheet environment. Further, the methods described herein may also be used wherever a recurrence can be used to describe the relationship(s) in the problem of interest, and the recurrences need not be linear; non-linear ones are accommodated just as easily in the spreadsheet. Examples which come to mind are simultaneous recurrences to model population dynamics or for evaluation of mutually recursive integrals. One thing is clear: we have merely scratched the surface, and the scope for exploitation of recurrences in Excel for the benefit of pedagogy is huge.

## 7 Acknowledgement

The authors wish to thank the anonymous reviewers for many very helpful comments and suggestions. The second author would like to acknowledge West Virginia University for the award D3-10-040 that supported this work.

## 8 Exercises

Unless otherwise stated, all rates are *nominal*, i.e., *per annum*. The rate per period must always be used; e.g., 8% nominal, compounded quarterly is 2% per quarter.

1. **FV.** Helen borrows \$6,000 agreeing to repay the principal with interest at 3% compounded semi-annually. What does she owe at the end of 4 years?
2. **FV.** John put \$50,000 into a savings account at the birth of his son. If the account pays 2.2% compounded semi-annually, how much will it contain when the son is 18 years old?
3. **FV.** David's piece of land worth \$750,000 will, it is estimated, increase in value each year by 4% of the previous year's value for 12 years. What will be its value at the end of this time?

4. **PV.** I owe \$12,500 due without interest 3 years from today. What sum should my creditor, Natasha, be willing to accept today if she is able to invest money at 4%, compounded six-monthly?
5. **PV.** In buying a house, Eric pays \$100,000 cash and agrees to pay \$175,000 two years later. At 6% compounded semi-annually, find the cash value of the home.
6. **PMT.** Wasim borrows \$50,000 today with interest at 5% compounded semi-annually. He agrees to pay \$10,000 one year from today, \$20,000 two years from today and the balance 3 years from today. Find the final payment \$X.
7. **PMT with PIVOT.** If money is worth 6% effective, what equal payments \$X at the end of 1 year and 3 years will equitably replace Tom's obligations: \$120,000 due in 3 years without interest and \$400,000 with interest from today at 4% compounded semi-annually due in 6 years?
8. **PVA.** Jenny's lot is sold for \$500,000 cash and \$250,000 per annum for the next 4 years. If money is worth 6% effective find the cash value of the lot.
9. **PVA.** Find the cash equivalent of a Bill's annuity paying \$10,000 at the end of each month for 15 years, assuming money is worth 12% compounded monthly.
10. **PV.** It is estimated that a stand of timber will net \$150,000 a year for the next 10 years and that the land can then be sold for \$1,500,000. Find a fair price today, money worth 5%.
11. **RATE.** Keith receives \$12,500 trade-in for his old car when buying a new model costing \$47,500. If the balance can be liquidated by payments of \$1,250 at the end of each month for 48 months, what monthly rate of interest is he being charged?
12. **NPER with RESIDUAL.** A widow as beneficiary of a \$5,000,000 insurance policy will receive \$1,000,000 immediately and \$200,000 every 3 months thereafter. If the company allows interest at 8% compounded quarterly, (a) how many full payments will she receive, (b) what additional sum paid with the last full payment will exhaust her benefits, (c) what sum paid 3 months after the last full payment will exhaust her benefits?

## 9 Answer to exercises

See also the downloadable Excel workbook containing a solution model, one problem per sheet.

1.  $\$6,000(1.015^8) = \$6,758.96$
2.  $\$50,000(1.011^{36}) = \$74,133.01$
3.  $\$750,000(1.04^{12}) = \$1,200,774.16$

4.  $\$12500(1.02)^{-6} = \$11,099.64$
5.  $\$100000 + \$175000(1.03^{-4}) = \$255,485.23$
6.  $X = 50000(1.025^6) - 10000(1.025^4) - 20000(1.025^2) = 25,934.04$
7.  $\$257,080.48$
8.  $\$1,366,276.40$
9.  $P = \$1,000,000(1 - 1.01^{-180}) = \$833,216.64$
10.  $P = \$2,079,130.12$
11. 1.3709%
12. .
  - (a) 25 full payments
  - (b)  $\$156,364.03$
  - (c)  $\$159,491.31$

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