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BOND UNIVERSITY
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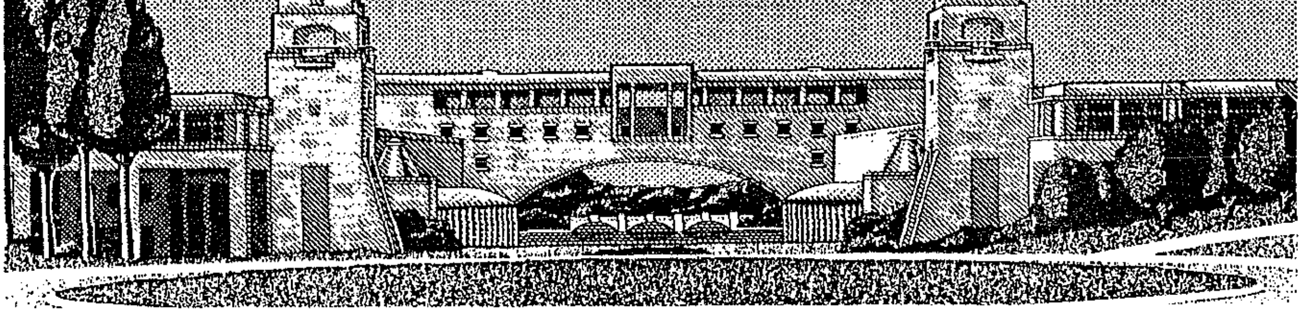

B O N D U N I V E R S I T Y

Exchange Rate Volatility, Monetary Policy Adjustment and Price Hysteresis

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Abstract

Nominal exchange rate volatility has been greater than that of "fundamentals" supposed to establish the exchange rates. A major contribution to our understanding of this volatility was given by Rudiger Dornbusch in his 1976 paper "Expectations and Exchange Rate Dynamics", where stickiness of goods prices forced exchange rates to carry all the short-run adjustment of the economy in response to unanticipated monetary shocks. The price stickiness was assumed, not explained.

Some recent authors, especially Baldwin and Krugman in the US and Bean in the UK, have linked modern industrial organisation theory to exchange rate behaviour to conjecture that the very volatility of exchange rate movements induces firms to have sluggish price adjustments. The concept that prices may respond differently to changes in other variables depending on the size of movement of those other variables as well as on their levels is called hysteresis or path dependence.

An immediate implication of price hysteresis is a renewed importance for economic policy to reduce exchange rate volatility if policy authorities expect exchange rate movements to redress severe current account imbalances. The size of exchange rate changes affects prices, which are normally the principal influences on current account adjustment.

This paper explores the linkage of exchange rate volatility and price movements within an exchange rate model incorporating monetary policy reactions. The model has already been used by the author to trace how policy reactions can themselves give rise to exchange rate overshooting. With the insights of the industrial organisation literature, we gain a better understanding of how closely monetary policy can affect exchange rate movements. In particular, we show how the parameters of the model providing the channel for monetary policy to cause "overshooting" of the exchange rate depend on perceived uncertainty about exchange rate movements, but in turn can change the level of the conditional variance of the exchange rate.

A Puzzle and a Partial Solution

Paul Krugman (originally 1987, published in 1989) offers a persuasive explanation for a significant puzzle in the behaviour of exchange rates and prices of traded goods. He observes that the large swings in nominal exchange rates experienced in the major western economies from the late 1970's to the 1990's have not produced appreciable effects on real variables in those economies. The volatility of the nominal exchange rates has been matched almost entirely by the volatility of real exchange rates, implying a sluggish reaction of prices for traded goods. Krugman suggests that the very volatility of the nominal exchange rates has in fact induced the muted reaction of real goods prices and consequently little adjustment in the volume of traded goods. This explains why floating exchange rates have not tended to eliminate substantial trade deficits for the US or surpluses for the Japanese, economies.

The argument that prices have not responded immediately to exchange rate movements is not new. Studies from a number of countries have shown that there is a slowness in price adjustment for imported goods following exchange rate movements. The size of the "pass-through" varies, but Goldstein and Khan's study (1985) indicates a lag of up to two years before the full adjustment takes place. Various explanations concentrating on costs of price adjustment and recognition lags by firms have been advanced to explain the lags. We also know that short- and long-run elasticities of import price adjustment differ: empirical evidence strongly supports this effect (Goldstein and Khan, 1985, pp.1090-1091), and the proffered explanation for the initial slowness in adjustment encompasses this effect—short-run adjustment costs are higher than long-run ones so in the long-run the degree of price adjustment will be more complete in response to a given unanticipated change in the nominal exchange rate.

Krugman's explanation gives a different reason for the slowness of price adjustment. It also shows why the response may change even without a lapse of time. He argues that the amount by which the prices adjust to exchange rate movements is linked to the size of the movements. The more volatile the exchange rate, the less the prices will be adjusted. This in turn means the trade account will adjust less since the prices affecting it are not very flexible. The exchange rate will be forced to adjust even more to attempt to bring the economy towards trade equilibrium. The volatility of the exchange rate is an indirect function of its own history in a reinforcing way.

Krugman's model is relatively informal and directly links import prices and general prices in the economy. Firms in the economy, competing internationally, "price to market". The basis for this activity is the sunk cost model Krugman and others have developed from recent research in industrial organisation. If firms have committed resources over time to obtaining market share, they will continue to operate to maintain that share even if the immediate impact is a loss. This reaction is particularly true for differentiated manufactured goods in which non-price competition is vigorous. These committed resources, or sunk costs, discourage a firm from exiting the market if losses are incurred in the short-run, and also discourage market entry from new firms that would tend to compete away abnormal profits if they are being made in the short-run. "This simple role of sunk costs can explain why small exchange-rate fluctuations might not have much effect on the pattern of specialization: A movement of a few percent in the relative cost of producing manufactured goods in the United States and in Germany will not make it worthwhile for firms to incur the cost of breaking into a new market, or induce firms already in the market to drop out" (Krugman, 1989, p.45). The persistence of market structure in response to changes of external forces such as exchange rate changes has been called market hysteresis, or path dependence.

The size of the US dollar exchange rate changes in the 1980's, however, has been more than small. Yet price adjustments and trade volume responses have still remained slight. Krugman completes the model to explain why even large adjustments of exchange rates might not induce much variation in trade flows. Perceived sunk costs may alter with the variability of the exchange rate movements. The simplest hysteresis model says that once the exchange rate has moved by a certain amount, entry and exit will occur in the market. The key insight by Krugman is that the margin within which the hysteresis occurs (the range of values the exchange rate must cross to induce an exit or entry decision) will vary with the amount of exchange rate variability observed in recent times. The history of the rate will be used to form expectations about future exchange rate changes. Greater recent variability will tend to imply almost as much future variability.

Thus a large appreciation of the exchange rate followed by a significant depreciation suggests that future appreciations will be succeeded by equally extensive depreciations. An entry or exit decision based on the appreciation might need to be reversed during the depreciation. Hence the next appreciation, even if large, may not be enough to induce (say) entry if it is believed that a depreciation inducing exit may follow. In this way, the size of the exchange rate change must be even greater to induce entry decisions.

Krugman offers a numerical example demonstrating how the expectational changes might occur for a firm. He uses an options pricing model since the decision to enter or exit is an option for the firm. The model emphasises the income effect of the exchange rate change: the decision to enter or exit a market is based on profits and losses (income) not on the setting of prices. The rest of Krugman's model, linking the exchange rate movements to prices that are adjusted sluggishly, is less formal.

Baldwin (1988) is critical of the Krugman approach in part for its informality but mainly for what he sees as a misplaced emphasis on the income-elasticity of demand for the firm's goods rather than on the import-price elasticity. He comments that the empirical evidence presents a puzzle about import-price sluggishness and that implies hysteresis in market entry and exit decisions. Krugman has not really explained the price sluggishness itself. The low level of exchange rate "pass through" from exchange rates to imported goods prices needs explanation, and Baldwin proposes it using the idea of path dependence in the firm's direct pricing decision.

The Exchange Rate Model

This paper demonstrates a further aspect of the circle linking exchange rate variability and pass-through parameters discussed by Baldwin and Krugman. The choice by monetary authorities of their policy reaction functions can affect the level of the exchange rate and the likelihood of its overshooting or undershooting (Papell (1984) and Adam (1986)). We now show that the choice of those functions can change permanently the conditional variance of the exchange rate and the ability of trade flows to provide adjustment for an economy in response to external shocks. It is possible that certain choices of policy parameters could substantially weaken the pass-through of the exchange rate into import prices and hence eliminate any opportunity for an economy to recover from significant trade account imbalances.

Our model of exchange rate dynamics between two trading countries allows for reactions from the monetary policy authorities. The model is based on Papell (1984) and Adam (1986), drawing from an earlier construct of Dornbusch (1976). It does not have a separate trade balance of Frenkel and Rodriguez (1982), but does allow exchange rate expectations to affect the setting of traded goods prices.

$$m_t - m^*_t - (p_t - p^*_t) = (y_t - y^*_t) - a_1(i_t - i^*_t) \tag{1}$$

$$y_t = \underline{y} + a_2(e_t + p^*_t - p_t) \tag{2}$$

$$y^*_t = \underline{y}^* - a_3(e_t + p^*_t - p_t) \tag{3}$$

$$(p_{t+1} - p^*_{t+1}) - (p_t - p^*_t) = a_4(e^e_{t+1} - e_t) + a_5(e_t + p^*_t - p_t) \tag{4}$$

$$m_t = \underline{m} + a_6 e^a_t + a_7(p^a_t - p^{*a}_t) + v \tag{5}$$

$$m^*_t = \underline{m}^* - a_8 e^a_t - a_9(p^a_t - p^{*a}_t) + v^* \tag{6}$$

$$i_t = i^*_t + (e^e_t - e_t) \tag{7}$$

In this model

- m is the logarithm of the domestic money supply;
- p is the logarithm of the domestic (traded good) price level;
- y is the logarithm of domestic real income;
- e is the logarithm of the exchange rate expressed as the domestic currency price of foreign exchange;
- i is the domestic nominal interest rate;
- v is a spherical random disturbance;
- e^e_{t+1} is the exchange rate value expected for period t+1 given the information available in period t;
- $*$ following a variable means that it refers to the foreign country;
- ^{a} following a variable means deviation from steady state;

$\underline{z}$ means steady state value of variable z.

As a monetary model of the exchange rate this construct contains only money in each economy. No interest-bearing assets are explicitly included. The money demand function for each economy, included in equation (1) above, responds to interest rate but no trade in interest-bearing assets takes place. As has become standard in these models, the money market equilibrium condition is expressed as the difference of the logarithms of the money demand functions. Analytical tractability is afforded by assuming equality across countries of interest rate semi-elasticities and of adjustment elasticities.

The real income (also output in this model) functions in equations (2) and (3) show that output in each country deviates from its steady state level as deviations from purchasing power parity occur. In particular a depreciation of the real exchange rate $((e_t + p_t - p^*_t) > 0)$ encourages supply above steady state from the domestic economy $(a_2 > 0)$ and discourages the supply in the foreign economy $(-a_3 < 0)$.

In our model, a critical variable will be a_4, the coefficient of expected exchange rate changes in the price dynamic equation. The logic of the price dynamic equation arises from a commentary by Michael Mussa in an early paper (1982) in which he noted that "The correct specification of the rule for the adjustment of the price of domestic goods is a matter of considerable importance. The standard assumption is that this price responds with some positive speed to the excess demand for domestic goods. In an economy system in which equilibrium prices are anticipated to be changing over time, this simple price adjustment rule is demonstrably inadequate. It is essential to supplement this rule with a term that adjusts p to expected changes in its own equilibrium value." (Mussa, 1982, p.90).

If we inspect the price dynamic equation given above, we see that the inflation rate (the left hand side) is driven by the expected change in the nominal exchange rate that in the final solution is central to the price adjustment mechanism, and by the deviation between the landed price of the traded good and the price charged by the domestic producers. The latter term represents the switching effect of excess demand, with a rise in the landed price (foreign price adjusted by the exchange rate) tending to enhance demand for the locally produced version and hence bid up the local price compared to the foreign price. An anticipated depreciation (rise in e) also drives local demand towards local production with similar effects on local price, because a depreciation will increase the price of foreign goods.

Equations (5) and (6) are monetary policy reaction functions, determining the supply of nominal money in each country according to an historical level and deviation of the exchange rate and relative price levels from their steady state values. The coefficients a_6, a_7, a_8, and a_9 are established by the monetary authorities in the two countries. If $a_6 > 0$, the domestic monetary policy authority accommodates exchange rate depreciation: a depreciation of the actual exchange rate above its steady state (so $e^a > 0$) induces the authority to increase the domestic money supply. A negative value for a_6 is "leaning against the wind", or offsetting, the exchange rate movement. Responses of the money supply to relative price deviations from steady state can also be characterised as accommodating (as a_7 or a_9 are positive) or offsetting (when a_7 or a_9 are negative).

Equation (7) is simply the uncovered interest parity relation that we assume to hold.

A solution for the model can be obtained by ignoring the error terms and assuming perfect foresight (so that $e^e_{t+1} = e_{t+1}$), substituting (2), (3), (5), (6) and (7) into (1), simplifying and writing as deviations from steady state, and combining with (4) to give the system of equations

$$Dz_t = \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix} (z_t - \underline{z}) \tag{8}$$

where

$$z_t = \begin{pmatrix} e_t \\ p_t - p^*_t \end{pmatrix}$$

$$Dx_t = x_{t+1} - x_t$$

and

$$d_1 = [(a_2 + a_3) - (a_6 + a_8)]/a_1$$

$$d_2 = [1 - (a_2 + a_3 + a_7 + a_9)]/a_1$$

$$d_3 = a_4[(a_2 + a_3) - (a_6 + a_8)]/a_1 + a_5$$

$$d_4 = a_4[1 - (a_2 + a_3 + a_7 + a_9)]/a_1 - a_5$$

In this solution we find that the response of the exchange rate to shocks or policy changes depends critically on the values of a_6, a_7, a_8 and a_9 selected by the monetary policy authorities in each country. As shown in Papell (1984) and Adam (1986), the extent to which the nominal exchange rate changes reacting to exogenous shocks can be characterised as overshooting or undershooting as the relative signs of the terms d_1, d_2, d_3 and d_4 change with different values of the a terms. Those papers give the full range of solutions under different assumptions made about the money demand functions.

Monetary Policy and Price Hysteresis

Suppose we restrict our discussion to the case of $D\underline{z} = 0$. This has the effect of setting a benchmark solution, one in which the overall changes of the exchange rate and price level are held at zero as movements in the variables offset each other around the steady state. This provides us with two equations for analysis, but we consider only the intersection of the two functions and solve for the equilibrium exchange rate in terms of the price variations and the level of exogenous uncertainty in the model. The solution is

$$e_t - \underline{e} = \varnothing_1(q_t - \underline{q}) - \varnothing_2(v_t - v^*_t) \tag{9}$$

where

$$q_t = p_t - p^*_t$$

$$\varnothing_1 = [d_2(a_4 - 1) - a_5)]/[d_1(1 - a_4) - a_5]; \ \varnothing_2 = a_4/[a_1(d_1(1 - a_4) - a_5].$$

We are interested in the variance of the exchange rate about the steady state level. In this formulation, the expression $(q_t - \underline{q})$ is predetermined at time t given the structure of the equations in (8); the unanticipated shock $(v_t - v^*_t)$, however, is not known at the start of time t and so its variation represents the variance in e. The variance is then

$$Var(e_t - \underline{e}) = E(e_t - \underline{e} - (E(e_t - \underline{e})/\Omega)) = a^2{}_4/[a_1(d_1(1 - a_4) - a_5)]^2 \, s^2{}_{v-v^*} \tag{10}$$

with the expectations operator E being take with respect to the information set Ω given at the beginning of the period t, and $s^2{}_{v-v^*}$ representing the unconditional variance of the combined error term $(v_t - v^*)$.

We are interested in the response of $Var(e_t - \underline{e})$ to changes of a_4. The partial derivative of the conditional variance with respect to the coefficient yields

$$\frac{\partial Var(e_t - e)}{\partial a_4} = \frac{2a_1a_4[d_1(1 - a_4) - a_5](a_1d_1 - a_5)}{\{a_1[d_1(1 - a_4) - a_5]\}^4} \cdot s^2{}_{v-v^*} \tag{11}$$

This expression may be positive or negative. The range in which it is negative is of most concern to us, as this implies a decline of a_4 will increase the conditional variance of the nominal and real exchange rate. The necessary and sufficient condition for the derivative to be negative is that $[d_1(1 - a_4) - a_5](a_1d_1 - a_5)$ be negative. If $(1 - a_4) > 0$, then sufficient conditions for the negative derivative are

$$a_2 + a_3 - (a_1a_5)/(1 - a_4) < a_6 + a_8 < a_2 + a_3 - a_5 \qquad if\ (a_1/(1 - a_4)) > 1 \tag{12}$$

or

$$a_2 + a_3 - a_5 < a_6 + a_8 < a_2 + a_3 - a_1a_5/(1 - a_4) \qquad if\ (a_1/(1 - a_4)) < 1 \tag{13}$$

Since the coefficients a_6 and a_8 are selected by the policy makers whereas a_1, a_2, a_3, a_4, and a_5 are given, it is clearly within the power of the policy makers to create the negative impact of a on the conditional variance of the nominal exchange rate. If the policy choices do in fact fall in the regions noted above, then policy makers face a severe challenge in managing the economy, given the next step in our construct.

We borrow the structure of Baldwin (1988) to show how an assumption about purchase patterns of consumers can lead firms to incorporate the variability of exchange rates in their price adjustment process and so make a_4 responsive to changes in $Var(e_t - \underline{e})$. In Baldwin's model, the supply of goods from the overseas market monopolist is priced by maximizing the sum of expected discounted profits over time. The explicit maximand includes the exchange rate to convert constant marginal costs to local currency, and has a "sales" measure that is a function of current and lagged prices. When we find the Euler equation for the solution, its solution defines a function giving price in terms of the contemporary exchange rate and lagged price. With the differentiation required to define the pass-through derivative, we obtain this variable as a function of contemporary price and exchange rate and a density function of the one-period ahead exchange rate.

Baldwin suggests that the density of the one-period ahead exchange rate can be indexed in the manner indicated by Rothschild and Stiglitz (1970). This allows us to consider a rise in the index as an increase in noise for a random variable. Such an increase is called a mean preserving spread. The issue then is to determine how the elasticity of price with respect to the exchange rate alters as the index changes. Taking a linear function for the sales function, with a separable term in the lagged price, Baldwin shows that the elasticity of price with respect to the exchange rate declines as the index of noise increases. Details of Baldwin's demonstration are given in the Appendix.

Assuming that the distribution of the exchange rate can be characterised by two parameters, there is a one-to-one correspondence between a mean-preserving spread and the conditional variance of the exchange rate. Thus Baldwin shows in his model that the pass-through derivative is decreasing in the conditional variance of the exchange rate. Most studies assume this elasticity is constant.

The possibility that the pass-through elasticity might change with alterations in the variability of the exchange rate is an issue of major concern for economic policy making in this model of exchange rate behaviour. We have already established that the conditional variance of the exchange rate is a function of a number of model parameters including the reaction parameters of the authorities and our version of the pass-through coefficient a_4. If the authorities alter the parameters of their reaction functions, as they may if they adopt a new policy regime, then the conditional variance of the exchange rate alters. Baldwin's model shows that the parameter a_4 is an inverse function of the conditional variance of the exchange rate. This means that a rise in the variance decreases a_4. In turn we now find the fall in a_4 not only reduces the pass-through of exchange rate expectations to the import price level p, but increases the level of the conditional variance of the exchange rate.

This "vicious circle" of a change in variance feeding on itself moves the equilibrium of the exchange rate model. Clearly the choice by the authorities of their reaction functions has both immediate effects in changing the exchange rate (as demonstrated in Papell (1984) and in Adam (1986)) but can also adjust the equilibrium level to which the exchange rate might return after a shock. The more variable the exchange rate, the less the pass-through to import prices and consequently the trade volume adjustment associated with those prices. The policy makers now have an additional burden to carry: their choices can affect not only the level but also the variability of the exchange rate on a permanent basis.

Conclusion

We can summarise the key insights of this paper with a diagram (Figure 1). The vertical axis is the average pass-through elasticity, ranging from low to high up the axis. The horizontal axis represents the variability of the real exchange rate, also ranging from low to high. The work of Dornbusch (1976) pointed to potentially high variability of exchange rates resulting from the sluggishness of goods prices to adjust in response to an unanticipated monetary supply shock. The implied pass-through elasticity, however, was high, with immediate adjustment of the prices arising after the initial exchange rate change. The coefficient of the pass-through was also constant. Hence we can note Dornbusch's result in the upper right hand corner of the Figure.

Empirically, Goldstein and Khan (1985) pointed to low short-run elasticities but high long-run elasticities. The relative variability of the exchange rate did not seem to matter, so we note them about the middle of the range on the horizontal axis. For a comparison, the purchasing power parity result, giving constant real exchange rates but immediate high pass-through effects from exchange rates to traded goods prices, is located in the top left hand corner.

The work of Baldwin and this paper point to an inverse relation between the size of the pass-through and the variability of the exchange rate, and are so indicated. Krugman did not focus specifically on the pass-through elasticity, but his work obviously underlies the construct.

The contribution of this paper is to highlight the way in which a choice of policy reactions by the monetary authorities can move the economy about in the Figure. An inappropriate choice (given by the coefficients lying in the intervals of equations (12) and (13)) could create induce the inverse relation between exchange rate variability and the size of exchange rate pass-through. This then vitiates the ability of changes in the exchange rate to alter the trade balance and restore the economy to a stable equilibrium. The economy moves away from its previous equilibrium but does not return.

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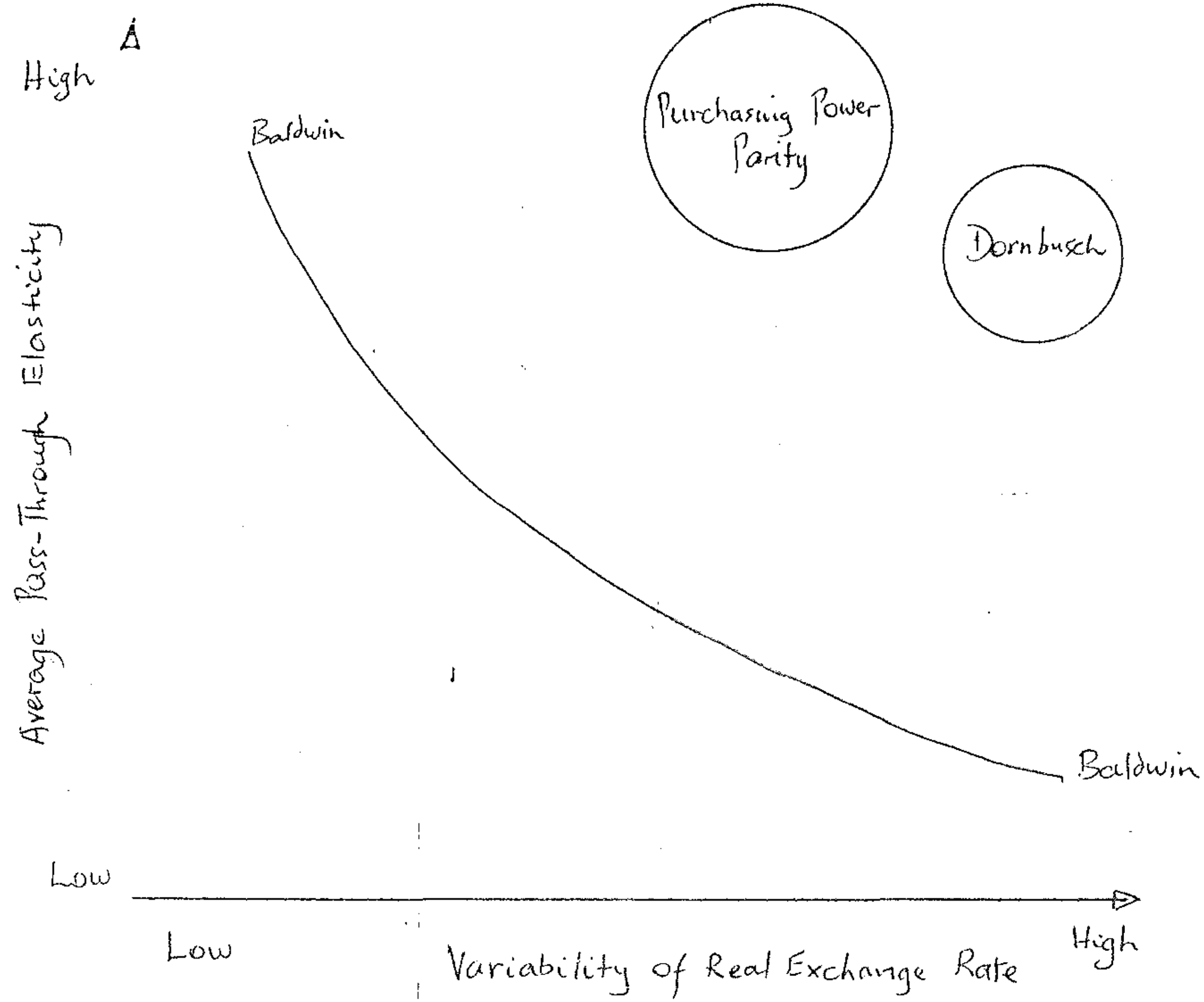

Figure 1: Representative Outcomes for Exchange Rate Variability and Average Pass-Through Elasticities

APPENDIX

from Baldwin & Lyons (See References)

In this paper we assume that current sales depend upon previous period price, in addition to the price charged in the current period. We fully recognize that this assumption is not the only possible choice, nor is it the most general. Nevertheless, it captures the basic effect of persistence in purchase patterns and greatly simplifies the analysis. Specifically, we assume that sales in period t+1, x_{t+1}, depend upon the price both in period t+1 and in period t:

$$x_{t+1} = x[p_{t+1}, p_t], \text{ for all } t. \tag{2.1}$$

We use an indefinite horizon setup in order to integrate the demand persistence effect into our macro model. Given equation (2.1) the problem of a foreign monopolist choosing the home-currency price of its sales to the home market is:

$$\max_{\{p_t\}} E\{ \sum_{t=0}^{\infty} \delta^t \pi[p_t, p_{t-1}, s_t] \}, \tag{2.2}$$

where,

$$\pi[p_t, p_{t-1}, s_t] = (p_t - s_t c_t)\, x[p_t, p_{t-1}].$$

Here s_t is the level of the nominal exchange rate, c_t is the constant marginal cost in foreign-currency terms. Assuming the firm takes as given the prices of all other goods as well as p_{t-1} and observes s_t before choosing p_t, the typical Euler equation for this problem is:

$$\frac{\partial \pi[\hat{p}_t, p_{t-1}, s_t]}{\partial p_t} + \delta E\{ \frac{\partial \pi[\hat{p}_{t+1}, \hat{p}_t, s_{t+1}]}{\partial p_t} \} = 0 \tag{2.3}$$

The expectation in (2.3) is conditioned on all information available at time t. The full solution to (2.2) would define a function which gives $\hat{p}_{t+i}$ (i=0,...∞) as a function of s_{t+i} and p_{t+i-1}. Consequently, in (2.3) $\hat{p}_{t+1}$ is a function of s_{t+1} and $\hat{p}_t$.

C. Exchange Rate Volatility and Pass-Through Behavior

The principal goal of this section is to determine the effect of a change in exchange rate volatility on the pass-through of exchange rates to import prices. Totally differentiating (2.3) with respect to p_t and s_t allows us to define the pass-through derivative, dp_t/ds_t, as a function of p_t, s_t and the density function of the one period ahead exchange rate, s_{t+1}. At time t, we assume that s_t is already known. The period t+1 exchange rate, however, is a random variable. A convenient way to represent an increase in exchange rate volatility is to consider a family of densities of s_{t+1}, $f[s_{t+1},r]$, where increases in the index r represent mean–preserving spreads (MPS). Rothschild and Stiglitz (1970) show that an MPS of the distribution of a random variable is equivalent to adding white noise to the random variable. Operationally, we differentiate the function which defines dp_t/ds_t with respect to the index r. The object then is to sign the derivative, $\frac{d(dp_t/ds_t)}{dr}$.

Since the first order condition involves first derivatives, the total derivative involves first and second derivatives. Consequentially, the derivative of the pass–through relationship with respect to r involves first, second and third derivatives of the period t demand function and period t+1 profit function. Unfortunately, there exists little economic reasoning that allows us to sign third derivatives. One way of dealing with this problem is to proceed with the formal analysis, simply assuming that the third derivatives have the signs necessary to produce the desired overall sign.

In this paper we take what we think is a more straightforward approach. We make assumptions on the functional form of the demand equation (2.1) that insures that all third derivatives are zero. Specifically we assume that the function is separable in p_{t+1} and p_t, and that the contemporaneous price enters linearly:

$$x_{t+1} = x[p_{t+1}]h[p_t], \text{ for all } t \tag{2.4}$$

where,

$$x[p_{t+1}] = a - bp_{t+1}, \text{ and}$$

$$h[p_t] > 0,\ h'[p_t] < 0,\ h''[p_t] < 0, \text{ and } h'''[p_t] = 0.$$

With these assumptions the pass–through derivative is:

$$\frac{dp_t}{ds_t} = \frac{c_t bh[p_{t-1}] + \delta h'[p_t]\frac{\partial}{\partial s_t}(E\Psi[s_{t+1}])}{2bh[p_{t-1}] - \delta h''[p_t]E\Psi[s_{t+1}]} \tag{2.5}$$

where,

$$\Psi[s_{t+1}] = \pi[p_{t+1},p_t,s_{t+1}]/h[p_t] = (p_{t+1} - c_{t+1}s_{t+1})x[p_{t+1}]$$

It is important to note that the $\Psi[s_{t+1}]$ function is convex in s_{t+1} since the profit function $\pi[s_{t+1}]$ is convex in s_{t+1} and $h[p_t]$ is positive.

For convenience we refer to the numerator and denominator of (2.5) as N and D. It is easy to show that both N and D are positive since we show below that $\partial E\Psi/\partial s_t$ is negative. Differentiating (2.5) with respect to r, we have:

$$\frac{d}{dr}\left(\frac{dp_t}{ds_t}\right) = D^{-2}\left(D\frac{d}{dr}\left[\delta h'[p_t]\frac{\partial}{\partial s_t}E\Psi[s_{t+1}]\right] + N\frac{d}{dr}\left[\delta h''[p_t]E\Psi[s_{t+1}]\right]\right) \tag{2.6}$$

The sign of (2.6) depends on the signs of the two terms $\frac{d}{dr}\left[\delta h'[p_t]\frac{\partial}{\partial s_t}E\Psi[s_{t+1}]\right]$ and $\frac{d}{dr}\left[-\delta h''[p_t]E\Psi[s_{t+1}]\right]$.

We turn first to the second term, $\frac{d}{dr}\left[\delta h''[p_t]E\Psi[s_{t+1}]\right]$. To evaluate this term we must investigate the distribution of s_{t+1}. Given our macro model, the ratio of the s_{t+1} and s_t is a random variable, $s_{t+1}/s_t = u$. Uncovered interest parity requires that the conditional expectation of u equals the ratio of the (gross) nominal rates of return on home and foreign currencies. Since s_t is pre–determined we can write $s_{t+1} = s_t u$. Recalling that $h'''[p_t]$ is zero, the second term can be written as:

(2.7) $$\delta h''[p_t] \int_{u=0}^{\infty} \Psi[s_t u] \frac{\partial f}{\partial r}(u,r)du.$$

The variable u cannot be negative since the exchange rate cannot be negative. To determine the sign of this term we apply standard Rothschild and Stiglitz (1970) techniques. Integrating (2.7) by parts twice, and using the facts that $T[0] = T[\infty] = 0$, and $T[x] \geq 0$, for $0 \leq x < \infty$, where $T(x) = \int_0^x \frac{\partial f}{\partial r}(z,r)dz$, we get:

(2.8) $$\delta h'' \int_0^{\infty} \Psi[s_t u] \frac{\partial f}{\partial r}(u,r)du = \delta s_t^2 h''[p_t] \left(\int_0^{\infty} \Psi_{uu}[s_t u] \left[\int_0^u f_r[z,r]dz \right] du \right),$$

Given that $T(x) \geq 0$, this term is negative since Ψ is convex in s_{t+1} and $h''[\cdot]$ is negative.

Next we address the first term in (2.6). The optimal p_t depends upon r as well as s_t so the first term in (2.6) can be written as:

(2.9) $$\frac{d}{dr}\left[\delta h'[p_t]\frac{\partial}{\partial s_t} E\Psi[s_{t+1}]\right] = \delta\left[h''[p_t]\frac{dp_t}{dr}\frac{\partial}{\partial s_t}(E\Psi[s_{t+1}]) + h'[p_t]\frac{\partial^2}{\partial s_t \partial r}E\Psi[s_{t+1}]\right]$$

By totally differentiating (2.3) with respect to p_t and r, we can define dp_t/dr. Noting that profits in period t+1 are convex in s_{t+1}, standard Rothschild–Stiglitz (1970) techniques can be uses to show that dp_t/dr is negative.

Since $h''[p_t]$ is negative, the sign of the first term on the right hand side of (2.9) depends on the sign of $\frac{\partial}{\partial s_t} E\Psi[s_{t+1}]$. The expectation here is conditioned on s_t, so this partial is equal to $dE(\Psi[s_{t+1}]|s_t)/ds_t$. Intuitively this is negative since according to our macro model if s_t increases, the s_{t+1} that is expected to occur is also higher. Since a higher exchange rate is detrimental to importer's profits, the expected profits should be revised downward when a higher s_t is observed. Formally,

(2.10) $$dE(\Psi[s_{t+1}]|s_t)/ds_t = \int_u \Psi'[s_t u]u\, f(u)\, du.$$

It is clear that (2.10) is negative since $\Psi'[s_t u]$ is negative and u and f(u) are non–negative.

The second term in (2.9) is negative, if $\frac{\partial^2}{\partial s_t \partial r}E\Psi[s_{t+1}]$ is positive. This expression can be written as:

(2.11) $$\int_u \Psi'[s_t u]u \frac{\partial f}{\partial r}(u,r)\, du.$$

By standard Rothschild and Stiglitz (1970) techniques, this integral is positive if the function $\Psi'[s_t u]u$ is convex in u. This in turn is true if $(2s_t \Psi''[s_t u] + us_t^2\Psi'''[s_t u])$ is positive. Applying our approach of assuming third derivatives are zero, this sum is positive since $\Psi[\cdot]$ is convex in s_{t+1}.

This finishes our demonstration that (2.6) is negative. In other words, the pass–through derivative is decreasing in the conditional variance of the exchange rate.