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The Euler Segment

Abstract
In this exercise, you will examine various features of the Euler segment using the scroll bar macro, a feature of Excel spreadsheets. The program "Euler Segment" is to be used in conjunction with the exercises.

Keywords
euler segment, orthocentre, circumcentre

Investigating the Euler Segment: Exercises using scrollbars on an Excel spreadsheet

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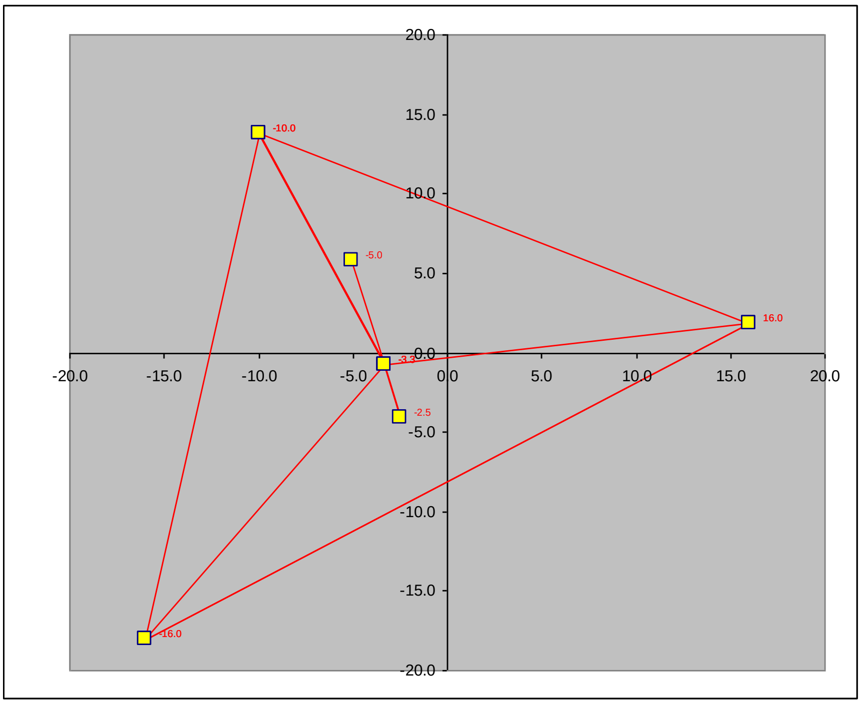

Background

The Euler segment of any triangle is the line segment between the triangle's orthocentre and circumcentre. It can be shown that the centroid is not only on this segment, but divides it in the ratio 2:1. The proof, which is discussed under Question 4, comes to us through Leonhard Euler (1707-1783), the most prolific mathematician in history.

The *orthocentre* is the point where the altitudes intersect. This may be inside or outside the triangle. The *circumcentre* is the point where the perpendicular bisectors of the sides of the triangle intersect. It too can be inside or outside the triangle. The *centroid* is the point where the medians intersect. The centroid, commonly called the center of gravity, can never escape the triangle's boundaries.

In the following exercises, you will examine various features of the Euler segment using the scroll bar macro, a feature of Excel spreadsheets. The spreadsheet [Euler Segment](#) is to be used in conjunction with the exercises.

Exercises

1. Open up the Excel Spreadsheet linked to this paper, called the "Euler Segment". The spreadsheet shows a triangle whose vertices are controlled by scroll bars. Clicking on the bar at either of its ends will cause the vertices to move left, right, up or down. By clicking and sliding the slide bar, these movements will be greatly enhanced. Practice for a while before moving to the next question.
2. The triangle contains the three special points mentioned above, namely, the circumcentre, the centroid and the orthocentre. Can you identify each of these?
3. Under what conditions will these three special points lie together? Move the vertices of the triangle around the plane using the scroll bars.
4. This question works through the proof that the circumcentre, centroid and orthocentre are collinear.

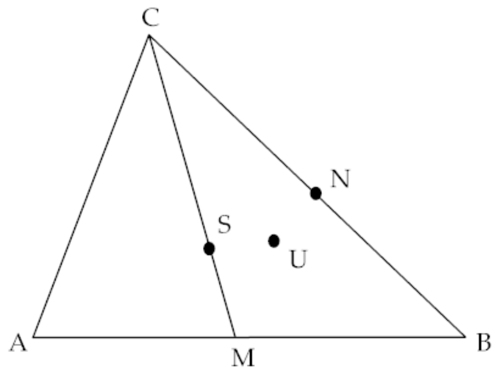

Figure 1: Diagram for Question 4

In the triangle ABC, shown in Figure 1, U and S are the circumcentre, and centroid of the triangle respectively. M and N are the midpoints of AB and BC respectively.

Construction:

Produce the line segment US to O so that SO = 2US. Join C to O.

Assumption:

We assume that the centroid S divides the median CM in the ratio 2:1.

Development:

(i) Prove that the triangles COS and SUM are similar.
(ii) Hence show that CO is an altitude of Δ ABC.
(iii) By similar argument, using the median AN, show that AO is also an altitude.
(iv) Hence show that O is the orthocentre of the triangle.
(v) Can you now state the theorem about the Euler segment UO?

5. Under what conditions will an extension of the Euler segment pass through one of the vertices of the triangle?
6. When the triangle changes shape, the circumcentre moves. When, where and why does this point move outside the triangle?

7. When the triangle changes shape, the orthocentre moves. When, where and why does this point move outside the triangle?
8. If the circumcentre is inside the triangle, can the orthocentre be outside the triangle?
9. Move the vertices to (0, 16.2), (15, -6), and (-15, -6). Verify that for this triangle the centroid cuts the segment in the ratio 2:1.
10. What connection is there between the circumcentre and the vertices of the triangle?
11. Move the vertices to (0, 0), (14, 0) and (4, 4). Scroll the vertex at (4, 4) so that it remains parallel to, and at a distance of 4 from the x-axis. What is the locus of the orthocentre describing? Can you determine the equation of this locus? What features of the locus can you determine?
12. Consider the triangle A = (0, 0), B = (t, 0) and C = (x_1, k) with k and t positive constants, as shown in Figure 2.

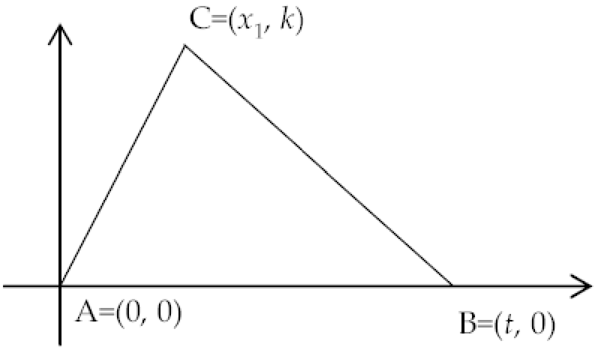

Figure 2: Triangle ABC and parameters

Prove that the locus of the orthocentre P for $x_1 \in \Re$ is the parabola $y = \left(\frac{t}{k}\right)x - \left(\frac{1}{k}\right)x^2$. Further show that when $t = k$, the parabola's latus rectum is the line segment AB.

13. If three forces of equal magnitude act at a single point the net force is given in magnitude and direction by the Euler segment as shown in Figure 3.

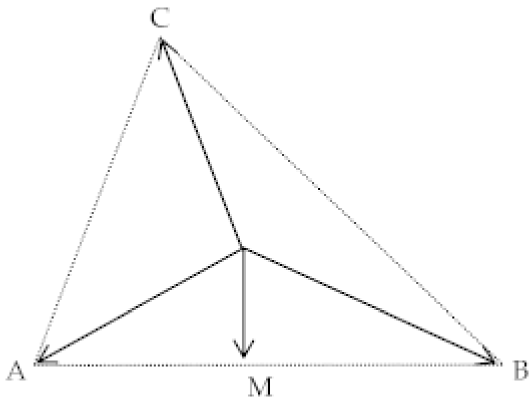

Figure 3: Three equal forces acting at a single point

Consider the following.

(i) Label the point where the forces act U. Consider the three vectors as having equal magnitude.

(ii) Place the point O, the orthocentre of the triangle, inside the triangle. (Estimate its position)

(iii) M is the midpoint of AB. Show that $\vec{UM} = \frac{1}{2}(\vec{UA} + \vec{UB})$

(iv) Construct vector $\vec{CO}$

(v) From question 4, show that $\vec{CO} = 2\vec{UM}$

(vi) Hence show that $\vec{UA} + \vec{UB} + \vec{UC} = \vec{UO}$

Using the spinners, try to examine the Euler segment as the net vector from the three vectors emanating from the circumcentre. Try looking at right angle triangles, isosceles triangles first. Write down your observations.

Solutions

1. Individual practice
2. Identification
3. Equilateral triangle
4. By considering the ratios of corresponding sides, and that the angles CSO and USM are vertically opposite, the triangles are similar. Since the angles SCO and SMU are equal, CM is a transversal on parallel lines. Since UM is perpendicular to AB, so also is CO, and hence CO is an altitude. Then by considering ASO and SUN in the same way, AO is also an altitude. Therefore, O is the orthocentre. That is to say, in any triangle the median is the point on the segment OU that divides it in the ratio 2:1.
5. Isosceles triangle
6. The circumcentre "leaves" the triangle at the midpoint of the side of the triangle. The triangle is right angled at that time. This is a consequence of the theorem that angles in a semicircle are right angles.
7. The orthocentre "leaves" the triangle at the vertex containing the right angle. The altitudes of a right-angle triangle meet at the vertex.
8. No
9. By inspection
10. A circle, whose centre is the circumcentre of a triangle, can be drawn through all of its vertices.
11. $y=\left(\frac{7}{2}\right)x-\left(\frac{1}{4}\right)x^2$ Vertex (7, 121/4), x intercepts 0 and 14.
12. By considering the gradient of CB to be $k/(x_1-t)$ and thus the gradient of AP to be $(t-x_1)/k$, the equation of AP (namely $y=(t-x_1)x/k$) intersects $x=x_1$ when $y=(t-x_1)x_1/k$. Thus, the locus of the point P is given by $y=tx/k-x^2/k$ as required. From this, the parabola's focal length becomes $k/4$, and since the vertex becomes $(t/2, t^2/4k)$, the latus rectum lies on the line $y=(t^2-k^2)/4k$. This is of course the x axis precisely when $t=k$.
13. By addition of vectors and the fact that the vector UC is the sum of vectors UO + OC, the proof becomes straight forward.