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Discovering Pi and its Measurement Variation: A Collaborative Cloud Activity

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Abstract

Measuring the circumference and diameter of objects, such as coins and lids, allows students to discover pi (?) from the circumference-to-diameter ratio. From multiple measurements of these two variables and the calculation of the simple ratio, measurement variation can be examined in a variety of numerical and graphical approaches, plus the influence of both random and systematic errors can be explored and percent error using the known value of pi. A best-fit mathematical model can be examined where a plot of circumference as a function of diameter yields the value of pi from its slope. By using a common US standard object (4-inch PVC pipe connector), the accuracy and precision of the measurements can be evaluated. Additionally, utilizing a pre-built Google Sheets spreadsheet (https://goo.gl/oYVQzS), data is collected, graphs produced, and computations performed in a shared environment in the cloud via Google Drive. This allows online collaboration and comparison within the class and between classes and even institutions, all-the-while building a large data set. Since the diameter measurement requires measuring through the center of the object, the diameter measurements could be underestimated which would yield a positive percent error in the determination of pi by this method. Random error should also be minimized as the number of objects measured (sample size) increases as well.

Keywords

online collaboration, Google Sheets, measurement error, cloud-based application

1. Introduction

Discovering the value of pi (π) by the circumference-to-diameter ratio is a common mathematical educational activity. The measurements of the circumference and diameter are accomplished in a variety of ways on a collection of circular or cylindrical objects. This article presents a modification of an activity posted by Curts [1] where class data was entered into a shared Google Sheets spreadsheet followed by analysis. We enriched the activity by examining measurement variation in numerical and graphical modalities, deriving pi by the circumference-to-diameter (C/D) ratio and seeing how it behaved over time as more measurements were accumulated, plus determining a mathematical model from a plot of circumference as a function of diameter. This was accomplished using a pre-built spreadsheet in Google Sheets; and sharing it with other classes allowed the amassing of a greater data set plus allowing comparison of individual measurements, and class-to-class measurements within in a school or between schools. If synchronous data entering by a variety of groups were performed, use of the chat function would foster online discussion. For novice learners, an instructor guided discussion is recommended [2]. Online collaboration can be enhanced using Google Sheets in a variety of cloud computing student activities [3].

Since the value of pi is known to many decimal places, percent error can be evaluated to judge accuracy, i.e. closeness to the known value of pi to 14 decimal places in Google Sheets. Additionally, multiple trials allows exploration of precision, i.e. measurement reproducibility; and also examination of possible causes of errors in the measurements. These errors can be random, or systematic when a bias occurs in the measurement. Further discussion of the types of errors and their behavior can be found in [4]. To explore the accuracy and precision of the measurement of circumference, diameter, and resulting computation of pi; a common US object, a 4-inch PVC pipe connector, was measured multiple times and the data amassed and analyzed.

The objectives of this activity are listed below.

- Measure the diameter and circumference of an object to the nearest 0.1 cm, enter data into spreadsheet, and examine the variation of the circumference-to-diameter ratio as measurements are added to the spreadsheet. Computations and plotting of data are all done automatically.
- Explore the plot of circumference as a function of diameter. A best-fit line is given on the plot as well as the equation of the line. The slope of the line yields

the value of pi. A value of r-squared, a measure of the goodness-of-fit of the line to the data, is also given.

- Compare the computation of pi presented for the two different methods explored above, along with the percent error.
- Examine behavior of any errors such as bias, random, etc. and postulate possible causes.
- For a common object, evaluate the accuracy and precision of the measurements and determination of pi.
- Explore the influence of rounding the data to the nearest 1 cm on accuracy and precision.
- For the pooled data, compare and discuss differences and/or similarities.
- Examine some elements of quality control.

Measurement instructions are provided on the "Directions" tab of the spreadsheet in Google Sheets. The following is the link to access the spreadsheet - https://goo.gl/oYVQzS and to get your own copy that can be edited go to File > Make a copy after accessing the spreadsheet.

This activity gives students an introduction to metrology, the science of measurement. It also helps get students on the path to developing a mindset to consider error when performing measurements. Instructors can modify this spreadsheet to fit their classroom needs and educational level in math and/or science from middle school through college. Students would need an understanding of the equation of a straight line.

Beware that Notes (black triangle in upper right corner of spreadsheet cell) are like comment boxes in Excel and are used to provide a vast amount of information, especially definition of terms, on the various tabs.

2. Experimental Measurements and Logistics of Pooling Data

Instructors will need to supply a variety of circular or cylindrical objects for students to measure such as coins, lids, tubes, etc. A 30-cm ruler (or meter stick) and a two-meter cloth measuring tape are recommended per student or group. Both need to be marked in millimeters, so that students can read measurements to the nearest 0.1 cm (1 mm). Using the ruler, measure the diameter, longest distance across the object, and record distance. Wrap the cloth tape measure around the perimeter or circumference of the object and record distance. Detailed instructions are provided on the "Directions" tab of the spreadsheet. Enter data into the “Data & Variation” tab of the provided Google Sheets spreadsheet which has been downloaded into the instructor’s Google Drive and shared with students in the “can edit” mode. Sharing the link to access the spreadsheet can be done by emailing it to students or posting it on a class website.

If pooling with other classes is desired, instructors may want to copy and rename the spreadsheet and assign a class ID for collecting the schoolwide pooled results. Each instructor can copy and paste their class data into the master pooled file. This allows the comparison of each instructor’s class with the pooled data. Figure 1 shows the logistics of pooling the data. This allows class-to-class comparison which can be made competitive by comparing percent error for accuracy and/or examining r-squared for the linear model for precision. The pooling of classes is essentially done by copy and paste into a blank copy of the renamed original file.

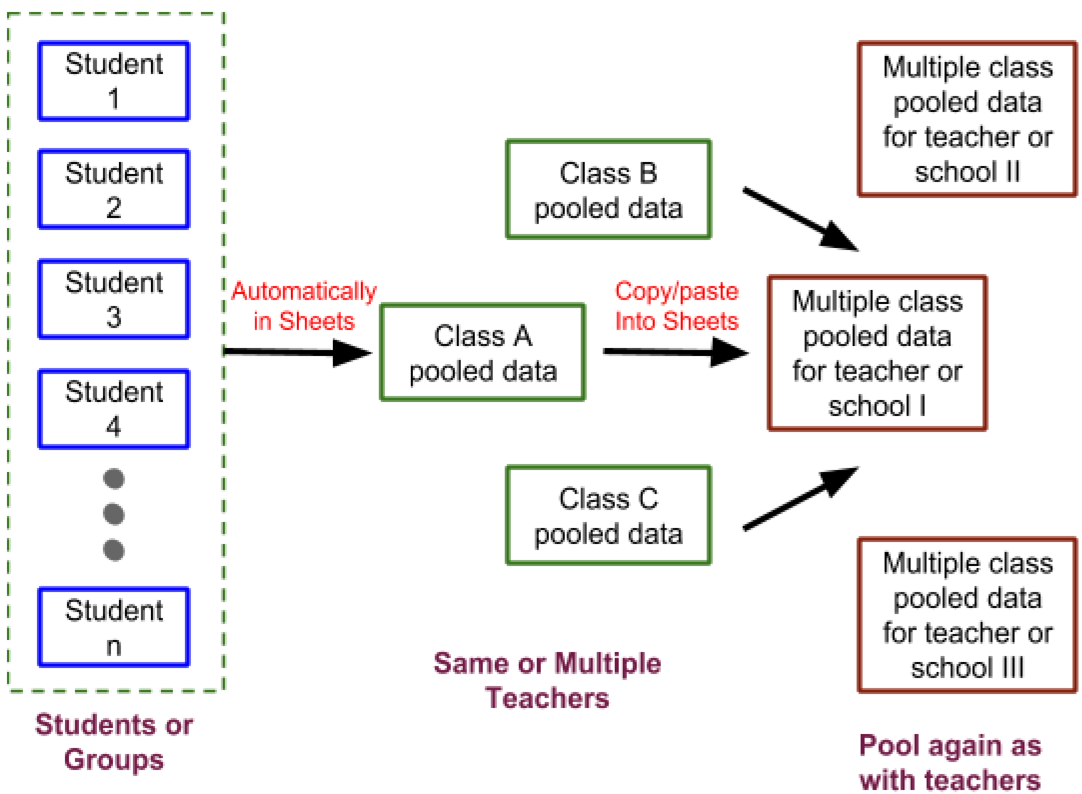


Figure 1 - Logistics of Pooling Data

3. Pre-built Spreadsheet

The pre-built spreadsheet will compute and graph results as they are entered by students. The "Data & Variation" tab (Figure 2) is where students enter data for the various measured objects. Within this tab computation of the circumference-to-diameter (C/D) ratio along with an average, running average, range, and standard deviation are calculated; these are all defined on the spreadsheet via notes. The individual values of the C/D ratio along with the running average are plotted as a function of trial or student number. The individual C/D ratio values illustrate the variation from measurement error (either high or low values), while the running average which uses more data shows results converge on or near the actual value of pi.

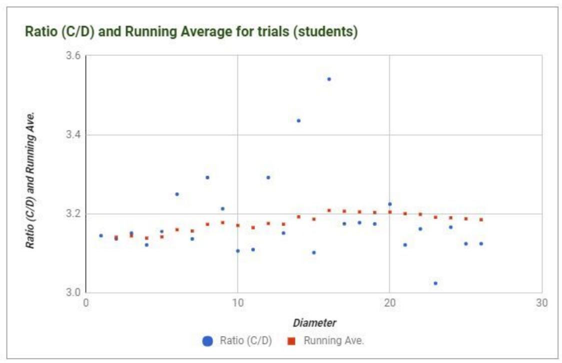
Collection of Data and Examining Variation

Student	Diameter	Circumference	Ratio (C/D)	Running Ave.	Class ID
1	11.7	36.8	3.1453	--	
2	10.2	32.0	3.1373	3.1413	
3	12.9	40.5	3.1518	3.1448	
4	4.1	12.8	3.1220	3.1391	
5	7.7	24.3	3.1558	3.1424	
6	10.0	32.5	3.2500	3.1604	
7	10.2	32.0	3.1373	3.1571	
8	11.3	37.2	3.2920	3.1739	
9	7.5	24.1	3.2133	3.1783	
10	10.3	32.0	3.1068	3.1712	
11	11.8	36.7	3.1102	3.1656	
12	11.3	37.2	3.2920	3.1761	
13	7.9	24.9	3.1519	3.1743	
14	6.2	21.3	3.4355	3.1929	
15	11.7	36.3	3.1026	3.1869	
16	3.7	13.1	3.5405	3.2090	
17	11.4	36.2	3.1754	3.2070	
18	10.1	32.1	3.1782	3.2054	
19	4.0	12.7	3.1750	3.2038	
20	4.0	12.9	3.2250	3.2049	
21	4.1	12.8	3.1220	3.2009	
22	11.7	37.0	3.1624	3.1992	

No. of results	Average Ratio	Range	Std. Deviation
26	3.1855	0.5155	0.107372

Blue shaded cells contain formulas - Do not type in them!!! (protected)

Setup for data up to 300 students!



Are there any outliers in your data (data points way off from the average ratio)?

How does the running average behave as more data is added?

Figure 2 - Data & Variation Tab

A plot of the circumference as a function of diameter (Figure 3) allows students to visualize the relationship between the two measurements. A linear regression line is added to give the best-fit straight line, provide an equation or mathematical model, and value of r-squared which is a measure of the goodness of fit of the line to the data points. Knowledge of linear regression is not needed and r-squared is explained on the spreadsheet. In an ideal or errorless world, this is a straight line represented by the equation $y = mx + b$ with $b = 0$, and where the slope, m, is the value of pi. Any non-zero value for the y-intercept (b) is from measurement error.

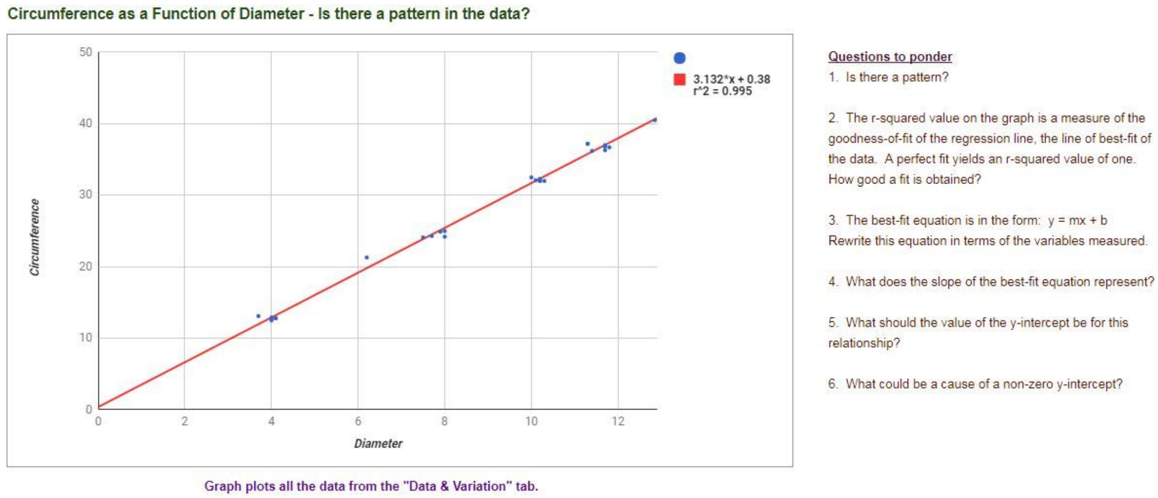


Figure 3 - Plotting the Data Tab

Simple random error, i.e. scatter in measurements, will yield a small value of the y-intercept in the model. As random error increases, it causes more scatter in the data points, randomly changes both the slope and y-intercept, and lowers the r-squared value. A sizable y-intercept, especially if the r-squared value is high (close to one), represents a systematic error most likely from the measurements [4, 5]. See the Ruler and Measurement Error interactive Excel spreadsheet to explore a variety of errors and visualize behavior [6]. As a note, using string as a replacement for the cloth measuring tape can cause a systematic error which influences the slope because of the possibility that the string will stretch.

Upon exploring the previous two tabs, the “Computing Pi” tab (Figure 4) examines the average of the C/D ratios and the slope of the C vs. D plot. The experimental value of pi is compared to the 14 decimal value used by Google Sheets in the PI() function as the reference or standard value (same as in Excel). This allows the computation of the absolute error (experimental value - reference value) and the percent error (difference x 100/reference value) which is the relative error. The sign of the difference and percent error reveals whether the experimental value is greater or lesser than the reference value and is helpful when trying to determine the cause of the error. Since the percent error for the mathematical model can be determined, the model can be validated if the error is small.

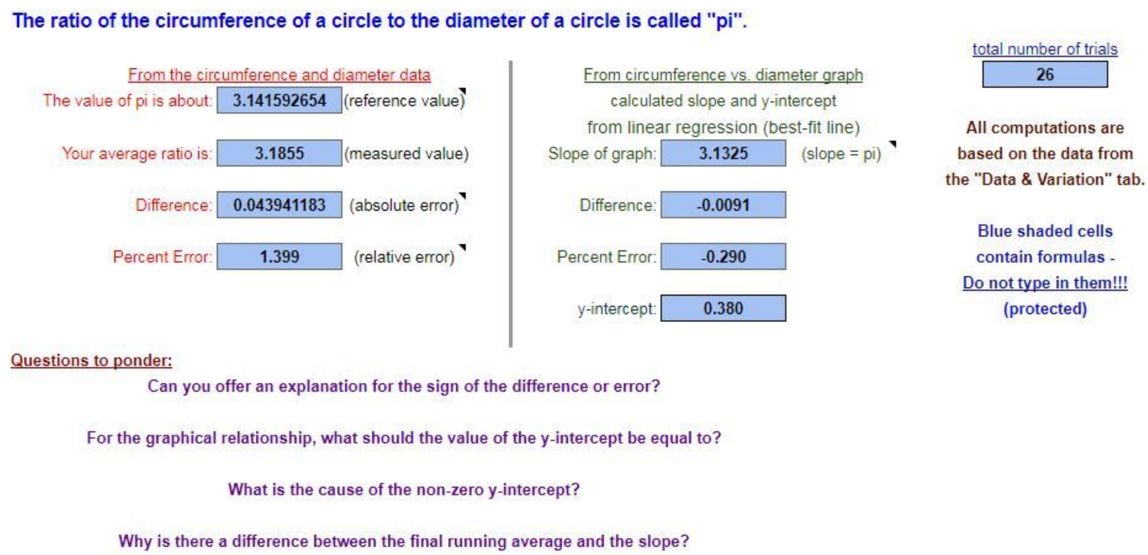


Figure 4 - Computing Pi Tab

To explore how good the two measurements are, we used a common standard object, the 4-inch PVC pipe connector, and collected ten replicate measurements which were made independently by ten different students. This allowed the calculation of the average diameter and average circumference, plus the percent coefficient of variation, %CV, or the precision can be evaluated and compared for the two measurements (Figure 5). The %CV's are comparable if means are different, while the standard deviations are not.

Since the C/D ratio yields the value of pi, the percent error or the accuracy can be evaluated. This allows any bias or systematic error to be determined. For the average percent error, the absolute value of the difference is used; hence, only the magnitude of the errors (all positive) are averaged. If the 4-inch PVC connector is used, we provide a link on this tab to pool the data.

Accuracy and Precision of Measurement

Let's examine the variation of the two measurements - the diameter and circumference.

Trial	Diameter	Circumference	Ratio (C/D)	%Error
1	12.9	40.4	3.1318	-0.3123
2	12.9	40.5	3.1395	-0.0655
3	12.9	40.4	3.1318	-0.3123
4	12.9	40.7	3.1550	0.4280
5	12.9	40.5	3.1518	0.3233
6	12.8	40.2	3.1406	-0.0308
7	12.8	40.6	3.1719	0.9639
8	12.8	40.5	3.1641	0.7152
9				
10				
average	12.9	40.5	3.1483	0.3939
std. dev.	0.05	0.15	0.01	--
%CV	0.4	0.4	0.5	--

Blue shaded cells have formulas in them and are protected.

Standard Object for Pooling Data
Use a 4 inch PVC pipe connector (see image below)

The Task
Pick one of the large circular objects (diameter > 10 cm) and have ten different students independently measure to 0.1 cm and record its diameter and circumference.

Examining the Results
The percent coefficient of variation, %CV, yields the variation in the measurement. The smaller the %CV, the less variation in the measurement.

- Precision
Which measured variable - diameter or circumference has the larger variation? Why (think cause of error)?

- Accuracy
Are the signs of the %errors random or is there a bias in one direction? If bias, why?

Graph plots correctly when data is added!

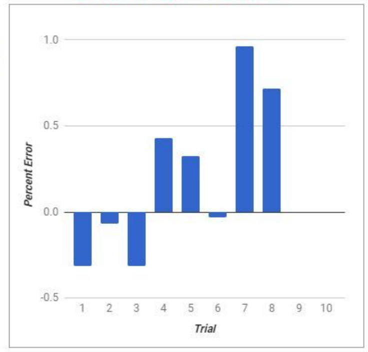


Figure 5 - Accuracy & Precision Tab

To give students an idea of why they are asked to read the ruler/tape measure to the nearest 0.1 cm or 1 mm, Figure 6 illustrates what happens when the 0.1 cm measurements are all rounded to the nearest 1 cm using the round function (=round (cell reference, decimal places). If rounded measurement values are used, students quickly see that both the accuracy and precision are degraded as both the percent error and %CV increase.

The Influence of Rounding Data

original data to 0.1 cm

Student	Diameter	Circumference	C/D Ratio
1	11.7	36.8	3.15
2	10.2	32.0	3.14
3	12.9	40.5	3.15
4	4.1	12.8	3.12
5	7.7	24.3	3.16
6	10.0	32.5	3.25
7	10.2	32.0	3.14
8	11.3	37.2	3.29
9	7.5	24.1	3.21
10	10.3	32.0	3.11
11	11.8	36.7	3.11
12	11.3	37.2	3.29
13	7.9	24.9	3.15
		mean	3.17
		std dev	0.07
	precision	%CV	2.06
	accuracy	%error	1.04

same data but rounded to the nearest 1 cm

Diameter	Circumference	C/D Ratio
12	37	3.08
10	32	3.20
13	41	3.15
4	13	3.25
8	24	3.00
10	33	3.30
10	32	3.20
11	37	3.36
8	24	3.00
10	32	3.20
12	37	3.08
11	37	3.36
8	25	3.13
	mean	3.18
	std dev	0.12
precision	%CV	3.80
accuracy	%error	1.18

The TASK:
Copy and paste 10-13 rows of diameter and circumference data from the "Data & Variation" tab into the yellow cells to the left. Right side will calculate!

How is the accuracy and precision influenced by rounding the data?

Can you suggest how via the measurements we could improve the accuracy and precision? Explain.

Figure 6 - Rounding Error Tab

4. Elements of Quality Control

Quality control, especially in manufacturing, is the production of a product to a specified standard or quantity, such as thickness of a metal sheet or length of a meter stick [7]. A typical quality control chart is the measurement tracked over time where the mean and standard deviation are computed to examine behavior of the individual values and plotted onto a classical Shewhart three sigma chart [7]. In this activity we chart the value of pi as if it were a product as a function of the trial (essentially time). Figure 7 is this plot which is generated automatically from the data collected on the "Data & Variation" tab.

The individual values are compared to the mean plus or minus one, two, and three standard deviations. Assuming a normal distribution of data points, 68.3% should be within plus/minus one standard deviation, 95.4% within two standard deviations, and 99.7% with three standard deviations. Normally, the plus/minus three standard deviations set the upper/lower control limits. Trials above or below the control limits are considered out of control, i.e. outliers in the data. Only two measurements (Figure 7) are outside of the mean plus/minus two standard deviations and they are both high values. High values of pi would be expected from diameter measurements that were not true diameter measurements (not through center of circular object) and hence low or short. In a statistics class, students could check the data set to see if it had a normal distribution. See [8] for instructions to accomplish this in Google Sheets.

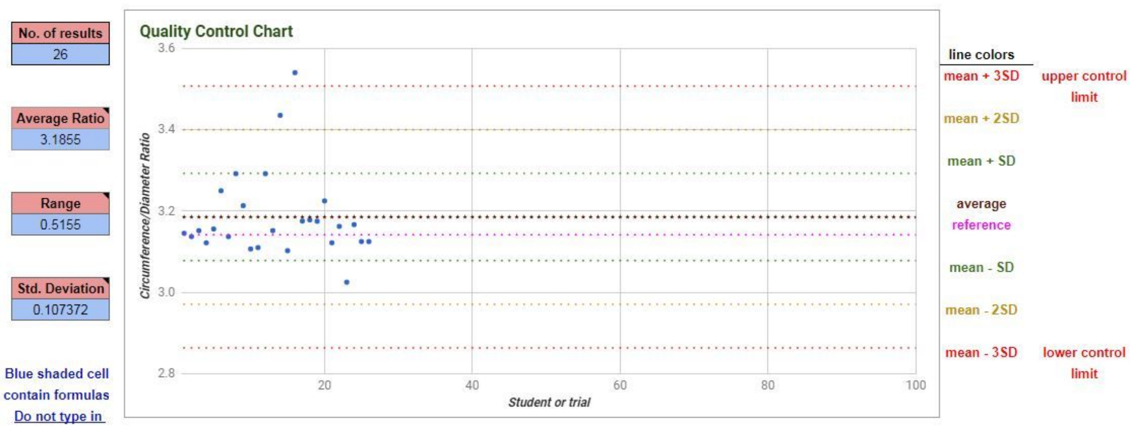


Figure 7 - Quality Control Chart (from Quality Control Tab)

5. User Feedback

This activity was piloted at a NASA Langley-Radford University Modsim workshop for high school mathematics teachers. Feedback was collected and is presented in Table 1. Comments were very positive and a selected sample given in Table 2. Results were very encouraging and the only drawback seemed to be it covered too much information; however, this could be corrected by modifying the spreadsheet to fit a particular class need; and as suggested by a teacher, tabs on the spreadsheet can be hidden if desired.

A couple of college statistics instructors were also very positive about using the spreadsheet.

6. Some Final Thoughts

Using a simple set of measurements, students discover how to determine the numerical value of pi, explore the accuracy and precision on the measurements plus examine variation in the measurements, both numerically and visually in graphical form and possible causes. A variety of metrics are given on the various tabs and instructors can tailor their instruction and spreadsheet use based on the class. Results are pooled in Google Sheets and comparisons for individual, the whole class, other classes, and total results can be made. If done in the synchronous mode, students can use the chat function available in Google Sheets to discuss the results and consider possible errors. Follow-up questions could be assigned as an online collaborative assignment to groups of students [9]. Having students master online collaboration is a valuable 21st-century workplace skill [10] and needs to be addressed in US undergraduate education [11] and, mostly likely, in the rest of the world as well.

Table 1 - Feedback from High School Math Teachers Workshop (n = 22 teachers)

Do you think this would be easy to use by students?					
1 strongly disagree 0	2 disagree 0	3 neutral 3 (13.6%)	4 agree 14 (63.6%)	5 strongly agree 5 (22.7%)	4.1 out of 5 86% agree
Would this be a good Pi Day activity, especially the collaboration between classes in your school? Could be a reward for most accurate and/or precise value of pi for a class!					
1 strongly disagree 0	2 disagree 0	3 neutral 3 (13.6%)	4 agree 12 (54.5%)	5 strongly agree 7 (31.8%)	4.2 86% agree
Is measurement variation (i.e. - dealing with error) an important concept for students to learn?					
1 strongly disagree 0	2 disagree 0	3 neutral 4 (18.2%)	4 agree 10 (45.5%)	5 strongly agree 8 (36.4%)	4.2 82% agree
What is the level of general overall learning? Think about your students using this activity!					
1 very low 0	2 low 3 (13.6%)	3 neither 6 (27.3%)	4 high 10 (45.5%)	5 very high 3 (13.6%)	3.6 59% high

Table 2 - Selective Teacher Comments

Positive comments
● I think this would be a great idea to do school wide. ● Since students just measure and input data, this would be more applicable in lower-level classes (for getting the data). Then a Statistics class could use the data and dig deeper into the calculations and evaluations. ● This will be a great activity for my Alg II trig classes...they will love the competition.
Constructive comments
● More of a higher level math activity at our school than algebra I or algebra II. ● It might be challenging to do class by class as a whole school, how do you distinguish which data came from which class? ● The students may first need some basic instructions on how and where to enter their data. Keeping the entries anonymous takes the anxiety away from those students who are fearful of entering incorrect measurements. The activity is great for teaching students the meaning of pi.

7. References

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