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The Use of Spreadsheets for the Seismic Design of Piles

Salah Sadek
Faculty of Engineering and Architecture, American University of Beirut, salah@aub.edu.lb

Fadi Freiha
Faculty of Engineering and Architecture, American University of Beirut, ffi01@aub.edu.lb

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The Use of Spreadsheets for the Seismic Design of Piles

Abstract
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Keywords
deep foundations, seismic design, soil-structure interaction, spreadsheet, civil engineering education

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Salah Sadek  
Faculty of Engineering and Architecture  
American University of Beirut  
salah@aub.edu.lb

Fadi Freiha  
Faculty of Engineering and Architecture  
American University of Beirut  
ff01@aub.edu.lb

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Abstract

Understanding the response of pile foundations to seismic loads is a necessary prerequisite to properly analyze and/or design them. This objective presents the civil engineering student and teacher with a complex problem, which typically challenges and strains the traditional didactic educational approach. In teaching the topic, we had faced significant difficulties in meeting the challenge of leaving the students with a thorough appreciation and understanding of the material. This was reflected in course evaluation results and comments. As such, an attempt was initiated to improve and enhance the learning/teaching process by incorporating a spreadsheet-based set of analysis/design tools with associated graphical representations. The approach described herein proved to be an effective and accessible method, according to the feedback solicited from students. In this paper, the basic background and available methods for the design of laterally loaded piles are presented along with the developed spreadsheet tool. Two design methods were included; one is a commonly used solution for laterally loaded piles; the other is a hybrid method which we compiled during the course of our work. The programming techniques and the methodologies adopted are also included. The Excel spreadsheets which were developed are presented along with illustrative examples.

Submitted March 2004; revised and accepted September 2004.

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1 Introduction

Understanding and mastering the seismic analysis and design of deep foundations is a challenging yet essential element of the advanced education of students in the field.
of civil engineering. Our past experience in the academic context of helping students achieve the desired outcomes had been a frustrating endeavor, given the time and effort invested. It is in part in response the need to provide a leaner and more efficient learning and teaching approach that the work described in this paper evolved.

In essence, the transfer of lateral loads from deep foundations to the subsurface strata is a complex soil-structure interaction problem. The movements and flexural stresses in the pile depend on the soil resistance, while the soil resistance is a function of the deformations of the pile itself. Furthermore, the ultimate resistance of a vertical pile to a lateral load and the deflection of the pile as the load builds up to its ultimate value are complex and involve the interaction between a semi-rigid structural element and soils which deforms partly elastically and partly plastically. Given the typically limited time and resources allocated to this topic in a three credit course, as other equally relevant applications are to be covered, imparting sufficient and fundamental understanding of this applied problem constitutes a real challenge that the spreadsheet approach presented herein attempted to meet.

In this paper, an Excel-based teaching and analysis tool is presented, which attempts to address some of the educational and applied challenges associated with the problem. The basic premise of the educational approach is anchored in the need and desire to empower the student to explore effects of the various parameters on the response of the piles in an environment designed to generate rigorous solutions with auto-generated and easy to interpret graphical representations. The paper also explores the benefits of coupling programming and spreadsheet calculations.

We have taken the spreadsheet developed through one cycle of testing in an actual classroom environment as part of a course on geotechnical earthquake engineering at the American University of Beirut in the Fall 2003 semester. The number of students registered for the course was 18 (10 graduate and 8 undergraduate students). The process and tool presented in this paper have significantly improved the educational experience from the teaching perspective, but also have generated more interest and response from the students than we had experienced in similar contexts using the traditional didactic approach. The use of the developed spreadsheets allowed us to adapt “what if?” scenarios to illustrate the effect and importance of certain parameters such as soil types, pile characteristics, levels and type of loading, etc. in the classroom in real-time.

2 Background—the engineering problem

Extensive work has been published on the resistance of piles to lateral loading. However, no simple design method has emerged, which can be universally applied to any soil and/or pile type. A number of interrelated factors influence the response of piles under seismic/lateral loading conditions. The pile stiffness is a critical factor given that it controls the deflection and determines whether the failure mechanism is one of rotation of a short rigid element, or flexure in bending of a long flexible member. The type of loading, whether sustained, alternating, or pulsating, influences the degree of yielding of the soil.

eJSiE 1(3):164–189
Seismic loading is typically considered as an additional “pseudo-static” lateral load applied to the pile. Piles are then analyzed by separating the two cases of vertical and lateral loading and then superposing the results. However, such an approach is inaccurate in non-elastic, non-linear media such as soils. Nevertheless, superposition still yields acceptable results given the fact that vertical loading involves the deeper layers of soil while the horizontal loading tends to involve the near-surface layers.

Earthquake loads are usually accounted for by the following two methods: Equivalent static load or equivalent dynamic load (Poulos and Davis [11], and Nair [10]). The need for an equivalent load arises if the solution of the rigorous problem is time-consuming and difficult.

In this paper, the presented analyses and design methods are typically based on the “pseudo-static” approach. Two methods are currently in use to determine the magnitude of the equivalent static load to be applied at the top of the pile (Nair [10]):

- The load taken is a certain percentage of the vertical static load.
- The total lateral load applied is taken to be the base-shear utilized in the seismic analysis of the building resting on the piles.

To obtain the base shear of the building or the horizontal force applied at the top of the pile, the formula $F_h = F_v S_a/g$ is used.

From attenuation curves, and using deterministic or probabilistic seismic hazard analyses, a peak horizontal acceleration is obtained. It is then corrected, based on the type of in-situ soil. Normalized response spectra are then used to assign the appropriate spectral acceleration $S_a$ for the design of the piles supporting the structure based on its natural period. $F_v$ is the vertical force applied on the pile and $g$ is the acceleration due to gravity.

### 3 The use of Excel spreadsheets

Despite the rational adoption of the simplified methodologies presented earlier, the analysis of the laterally loaded piles remains a difficult undertaking. The complexities are associated in part with the difficulty of calculating and visualizing the moments and deflections along the full length of the piles given certain loading conditions, as well as the understanding of the effects of the various inter-related parameters that influence the pile design (Figure 1). The “difficulty in visualizing behavior” was in fact the problem most identified by earlier student evaluation and feedback. As such, and in part in order to promote a more fundamental understanding of the problem, solutions and visualizations within an Excel spreadsheet environment were developed for use in the classroom. Whenever we needed to perform a task repeatedly, a Visual BASIC macro, which is basically a series of commands and functions that can be run on the fly, was embedded within the spreadsheet. This enabled us to automate some loops and repetitive procedures and solve for values using trial and error.
The seismic pile design methods which were incorporated in the spreadsheets were the “Characteristic Load Method” (Duncan et al [5]), and the “Hybrid Analytical Method” (based on Matlock and Reese [8] and Davison and Gill [4]).

These design methods were adopted given that they are typically used in engineering practice today. Automating the various expressions and procedures involved in the analyses allowed us to develop a reliable and useful design tool. In this context, when using this tool for education in the classroom, students are being introduced to professional level practice while at the same time relying on the same approach to gain knowledge of the fundamentals. The exposure to academic teaching tools which can also serve the purpose of real-world design aids is an added benefit which enriches the educational experience of the student of applied engineering.

### 3.1 Building the spreadsheets

A large set of parameters, some of which are interrelated, is involved in the analysis. In order to facilitate the interaction between the student and the spreadsheets, which may seem intimidating at first glance, the cell values that require an input by the user are highlighted in light yellow and the ones that will be automatically generated are highlighted in dark grey. Furthermore, the student is guided through the design-analysis process by comments/hints which are linked to the individual cells. They include the definition of each term, the units that should be used, etc. (Figure 2).

Some of the calculations involve trial and error procedures and loops. In such instances, Visual BASIC macros were assigned to specifically created buttons for the purpose of running these loops and obtaining the required parameter value with minimal effort. The idea is to lessen time spent on performing tedious and repetitive tasks which
encroach on the “quality-learning time” and reduce the tedium to a few mouse clicks while maintaining the advantages of spreadsheet interfaces (Figure 2). It is important to add here that the automated processes do not create “black boxes” invisible to the student, but merely perform those tasks that are fully understood and clearly explained.

After entering all the required parameters the response of the piles, expressed in terms of moments and deflections along the pile length, can be calculated at a depth increment assigned by the user. Charts are then auto-generated in separate worksheets to visualize the variation of these moments and deflections with depth. To do that, the usual charting procedure is used and recorded in a new macro, its code modified to fit regardless of the actual data limits. This specific macro is assigned to a button for future re-use. This ability of creating “instantaneous” graphed results as variables are modified or changed is at the core of the power of the spreadsheets in an educational context. Students were encouraged to explore the effect of changing the pile stiffness, material, soils, etc. on the predicted behavior. This was simply not possible prior to the introduction of this approach into the classroom.

Based on the feedback we received from the students who took part in the course evaluation, a majority noted the spreadsheets used in the seismic design of piles as key for their appreciation of the various elements involved and their contribution to the overall response. The spreadsheets enhanced the problem solving task extensively and allowed us to assign a greater number of different tasks and problems.

Figure 2: Light yellow cells, dark grey cells, comments, and VB buttons used in the various spreadsheets
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In the following sections the two analysis methods are illustrated through the example of a pile embedded in clay and supporting an earthquake load equivalent to a static horizontal load of 0.1 MN and a moment of 0.05 MJ. The response of the pile will be analyzed using the created Excel spreadsheets. Tables and figures are added to show the results of both methods allowing for some comparison.

The example considers a square concrete free-headed pile, 15 m long, 0.6 m wide. The concrete modulus is equal to 27800 MPa. The pile is embedded in a plastic, normally consolidated clay, with a linearly increasing soil modulus. The soil undrained shear strength is 48 kPa, the soil modulus is 36 MPa (= 750× undrained shear strength), and the coefficient of modulus variation is approximately 6000 kPa m\(^{-1}\) depth.

4 Characteristic load method (CLM)

The characteristic load method can be used to determine:

1. Deflections due to lateral loads for free-head conditions, fixed-head conditions, and the “flag-pole” condition.
2. Deflections due to moments applied at the ground line.
3. Maximum moments for the free-head conditions, fixed-head conditions and the flag-pole condition.
4. The location of the maximum moment in the pile or drilled shaft.

Despite the number of simplifying assumptions the characteristic load method still involves many parameters. Tabulating these parameters was deemed necessary in order to ease the spreadsheet-based trial and error processes (Figure 3). All relevant parameters are explained in detail in the following paragraphs.

4.1 Non-dimensional relationships with characteristic load \(P_C\) and moment \(M_C\)

The characteristic load method formulation is based on non-dimensional relationships related to load and moment. Equations (1)–(3) describe load and moment.

\[
P_C = \lambda_1 B^2 E R_1 \left( \frac{\sigma_p}{E R_1} \right)^{m_1} (\varepsilon_{\text{50}})^{n_1}\]
\[
M_C = \lambda_2 B^3 E R_1 \left( \frac{\sigma_p}{E R_1} \right)^{m_2} (\varepsilon_{\text{50}})^{n_2}\]
\[
R_1 = \frac{64I}{\pi B^4}\]

Note that the moment of inertia ratio, \(R_1 = 1.00\) for solid circular cross-sections and 1.70 or solid square cross-sections. For plastic clay and sand, \(\lambda_1 = \lambda_2 = 1.00\). For
Seismic Design of Pile Foundations

Table 1: Typical $\varepsilon_{50}$ values (Matlock [9])

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$\varepsilon_{50}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbed, remolded, soft clay</td>
<td>0.020</td>
</tr>
<tr>
<td>Normally consolidated medium clay</td>
<td>0.010</td>
</tr>
<tr>
<td>Brittle, stiff, sensitive clay</td>
<td>0.005</td>
</tr>
<tr>
<td>Medium dense sand with little or no mica</td>
<td>0.002</td>
</tr>
</tbody>
</table>

brittle clay, $\lambda_1 = (0.14)^{m_1}$, and $\lambda_2 = (0.14)^{m_2}$. For clay, $\sigma_p = 4.2s_u$. For sand $\sigma_p = 2C_{ps}\gamma B \tan^2 \left(45^\circ + \Phi'/2\right)$.

$Pc$ is the characteristic shear load, and $Mc$ the characteristic moment load. $\lambda_1$ and $\lambda_2$ are dimensionless parameters that depend on the soil stress-strain behavior. $B$ represents the diameter of the foundation, and $E$ the modulus of elasticity of the foundation (200,000 MPa for steel; 4700 $\sqrt{T_d}$ for concrete; 11,000 MPa for pine wood), where $f'_c$ is the 28-day compressive strength of concrete (MPa). The representative passive pressure of soil is denoted by $\sigma_p$, and $\varepsilon_{50}$ is the axial strain at which 50% of the soil strength is mobilized, obtained from triaxial compression tests or from Table 1. The exponents $m_1, m_1, m_2, n_2$ are those from Table 2. $I$ is the moment of inertia of foundation, and $s$ the undrained shear strength from the ground surface to a depth of $8B$. The effective friction angle (deg) from ground surface to a depth of $8B$ is represented by $\Phi'$, while $C_{ps} = \Phi'/10$ denotes the passive pressure factor. Finally, $\gamma$ is the unit weight of soil from ground surface to a depth of 8-pile diameters. If the water table is within this zone, use a weighted average of $\gamma$ and $\gamma_b = \gamma - \gamma_w$. All the above parameters were tabulated in Excel in the format shown in Figure 3. The soil properties are reduced to

Figure 3: The Excel worksheet of the characteristic load method

![Table 1: Typical $\varepsilon_{50}$ values (Matlock [9])](image-url)
Table 2: Values of the exponents $m$ and $n$ (Evans and Duncan [6])

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>For $P_c$</th>
<th>For $M_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m_1$</td>
<td>$n_1$</td>
</tr>
<tr>
<td>Clay</td>
<td>0.683</td>
<td>-0.22</td>
</tr>
<tr>
<td>Sand</td>
<td>0.57</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

Table 3: Constants for Load-deflection equations $y_t/y_c = a_1 (P_t/P_c)^{b_1}$ (Brettmann & Duncan [2])

<table>
<thead>
<tr>
<th>Constant</th>
<th>Free head</th>
<th>Fixed head</th>
<th>Free head</th>
<th>Fixed head</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>50</td>
<td>14</td>
<td>119</td>
<td>28.8</td>
</tr>
<tr>
<td>$b_1$</td>
<td>1.822</td>
<td>1.846</td>
<td>1.523</td>
<td>1.5</td>
</tr>
</tbody>
</table>

those of the upper 8-pile diameter depth.

Three main groups of non-dimensional relationships are used in the characteristic load method: Load-deflection, moment-deflection, and load-moment. Different relationships for each group are used for sands and clays. The load-deflection and load-moment relationships are further subdivided into free-head and fixed head conditions.

A single-term exponential equation in the form of $y = ax^b$ where $a$ is a constant and $b$ is an exponent, is used for the non-dimensional relationships. The coefficients $a$ and $b$ are obtained from a power regression analysis. The constants and exponents for each of the relationships are determined within the spreadsheet tool using an iterative, non-linear, least squares curve fitting technique.

\[
y_t/y_c = a_1 (P_t/P_c)^{b_1} \quad \text{With } a \text{ and } b \text{ deduced from Table 3.} \tag{4}
\]

\[
y_t/B = a_2 (M_t/M_c)^{b_2} \quad \text{With } a \text{ and } b \text{ deduced from Table 4.} \tag{5}
\]

\[
P_t/P_c = a_3 (M_{\text{max}}/M_c)^{b_3} \quad \text{With } a \text{ and } b \text{ deduced from Table 5.} \tag{6}
\]

In these equations, $y_t$ is the ground line deflection, $B$ the pile or drilled shaft diameter or width, $P_t$ the lateral load at top of pile or drilled shaft, $M_t$ the moment at the top of pile or drilled shaft, and $M_{\text{max}}$ is the maximum moment.

The flexural stiffness, $E_p I_p$, of reinforced concrete piles and drilled shafts should be reduced when tensile stresses are large enough to cause cracking. The magnitude of the reduction of $E_p I_p$ depends on the ratio of the maximum moment computed to the ultimate moment capacity or cracking moment of the section. For cracked sections, the
Table 4: Constants for Moment-deflection equations $y_t/B = a_2 (M_t/M_c)^{b_2}$ (Brettmann & Duncan [2])

<table>
<thead>
<tr>
<th>Constant</th>
<th>Clay</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2$</td>
<td>21</td>
<td>36</td>
</tr>
<tr>
<td>$b_2$</td>
<td>1.412</td>
<td>1.308</td>
</tr>
</tbody>
</table>

Table 5: Constants for Load-moment equations $P_t/P_c = a_3 (M_{max}/M_c)^{b_3}$ (Brettmann & Duncan [2])

<table>
<thead>
<tr>
<th>Constant</th>
<th>Free head</th>
<th>Fixed head</th>
<th>Free head</th>
<th>Fixed head</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_3$</td>
<td>1.22</td>
<td>1.63</td>
<td>0.425</td>
<td>0.669</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.79</td>
<td>0.86</td>
<td>0.762</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Moment of inertia of concrete is reduced to 40% – 50% of the value for the full-uncracked concrete section. No reduction is made in the moment of inertia of the steel.

4.2 Deflection due to combined load and moment

Given that the response is non-linear, simply adding the deflections caused by the lateral load and the moment is not sufficient. Instead, the non-linear effects should be taken into account by using a non-linear superposition procedure accounted for in the spreadsheet solution developed.

The first step in the non-linear superposition is to calculate the deflections that would be caused by the load $P_t$ or $E_4$ acting alone $y_{tp}$ and by the moment $M_t$ or $H_4$ acting alone $y_{tm}$ using the equations listed in the previous section and written in Excel as follows:

For $y_{tp}$ or $E_6$:

$y_{tp} = \text{IF}(\text{AND}(B4="sand",B21="free"),B17*119*((E4/E2)^1.523),\text{IF}(\text{AND}(B4="sand",B21="fixed"),B17*28.8*((E4/E2)^1.5),\text{IF}(\text{AND}(B4="clay",B21="free"),B17*50*((E4/E2)^1.822),\text{IF}(\text{AND}(B4="clay",B21="fixed"),B17*14*((E4/E2)^1.846),"--"))))$)

For $y_{tm}$ or $H_6$:

$y_{tm} = \text{IF}(\text{AND}(B4="sand",B21="free"),B17*36*((H4/H2)^1.308),\text{IF}(\text{AND}(B4="sand",B21="fixed"),B17*36*((H4/H2)^1.308),\text{IF}(\text{AND}(B4="clay",B21="free"),B17*21*((H4/H2)^1.412),\text{IF}(\text{AND}(B4="clay",B21="fixed"),B17*21*((H4/H2)^1.412),"--"))))$

The second step is to get the load $P_m$ that can cause $y_{tm}$ and the moment $M_p$ that can cause $y_{tp}$ as follows:

For $P_m$ or $E_6$:

$P_m = \text{IF}(\text{AND}(B4="sand",B21="free"),((H6/(B17*119))^(1/1.523))*E2,\text{IF}(\text{AND}(B4="sand",B21="fixed"),((H6/(B17*28.8))^(1/1.5))*E2,$

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IF(AND(B4="clay",B21="free"),((H6/(B17*50))^(1/1.822))*E2,
    IF(AND(B4="clay",B21="fixed"),((H6/(B17*14))^(1/1.846))*E2,"--")))

\[ M_p \text{ or } H_8 = \begin{cases} 
    \text{IF(AND(B4="sand",B21="free"),((E6/(B17*36))^(1/1.308))*H2,} \\
    \text{IF(AND(B4="sand",B21="fixed"),((E6/(B17*36))^(1/1.308))*H2,} \\
    \text{IF(AND(B4="clay",B21="free"),((E6/(B17*21))^(1/1.412))*H2,} \\
    \text{IF(AND(B4="clay",B21="fixed"),((E6/(B17*21))^(1/1.412))*H2,"--")))}
\end{cases}

The third step is computing the average of \( y_{tpm} \) caused by the load \( P_t + P_m \) and \( y_{tmp} \) caused by \( M_t + M_p \) using the equations of the previous section:

\[ y_{tpm} \text{ or } E_{10} = \begin{cases} 
    \text{IF(AND(B4="sand",B21="free"),B17*119*(((E4+E8)/E2)^1.523),} \\
    \text{IF(AND(B4="sand",B21="fixed"),B17*28.8*(((E4+E8)/E2)^1.5),} \\
    \text{IF(AND(B4="clay",B21="free"),B17*50*(((E4+E8)/E2)^1.822),} \\
    \text{IF(AND(B4="clay",B21="fixed"),B17*14*(((E4+E8)/E2)^1.846),"--")))}
\end{cases}

\[ y_{tmp} \text{ or } H_{10} = \begin{cases} 
    \text{IF(AND(B4="sand",B21="free"),B17*36*(((H4+H8)/H2)^1.308),} \\
    \text{IF(AND(B4="sand",B21="fixed"),B17*36*(((H4+H8)/H2)^1.308),} \\
    \text{IF(AND(B4="clay",B21="free"),B17*21*(((H4+H8)/H2)^1.412),} \\
    \text{IF(AND(B4="clay",B21="fixed"),B17*21*(((H4+H8)/H2)^1.412),"--")))}
\end{cases}

4.3 Maximum moments and their location

In fixed-head flexible piles or drilled shafts, the maximum moment occurs at the top, where the pile or shaft is embedded in the cap. In free-head flexible piles and drilled shafts, the maximum moment occurs at some depth below the ground. The magnitude and location of the maximum moment can be estimated using the theory, for soil modulus increasing linearly with depth: the characteristic length \( T \), solution by trial and error of the following equation is first calculated.

\[ y_{comb} = \frac{2.43 P_t}{E_p I_p T^3} + \frac{1.62 M_t}{E_p I_p T^2} \quad (7) \]

This is done by running a simple Visual BASIC macro assigned to a button named "Check". (Ref. Figure 3). This macro is as follows:

```
Sub trialanderror()
    Cells(19, 5) = 0
    Do
        Cells(19, 5) = Cells(19, 5) + 0.001
        Cells(21, 5) = Cells(20, 5)
    Loop Until Cells(21, 5) = "check"
End Sub
```

The formula in Cells(20,5) or E20 is:

\[ =\text{IF(AND(((2.43*E4)/(B23*B19))*E19^3)+((1.62*H4)/(B23*B19))*E19^2)/E13>0.99,)(((2.43*E4)/(B23*B19))*E19^3)+((1.62*H4)/(B23*B19))*E19^2)/E13<1.01)),"check",IF(OR(B21="Fixed",B21="fixed"),"check","retry")\]
SEISMIC DESIGN OF PILE FOUNDATIONS

The idea is to keep increasing the value of

\[
\left(\frac{2.43P_t}{E_pI_p}T^3 + \frac{1.62M_t}{E_pI_p}T^2\right) / y_{comb}
\]

from zero until it reaches 1. At that point the macro stops and the correct value for the characteristic length is given in cell E19.

Then the moment \( M_z \), at a depth equal to \( z \), is calculated from equations derived from the values in Table 6 and from equation (8).

\[
M_z = A_m P_t T + B_m M_t
\]

To use Excel, a formula needed to be created for \( A_m \) and \( B_m \). To do that, poly-fitting in MATLAB was used. Then, linear interpolation is used to get the correct \( A_m \) or \( B_m \) for a particular \( Z_{max} \). Two typical Excel formulas for \( A_m \) and \( B_m \) can be written as follows:

\[
A_m \text{ or } L12 = \text{IF}(\text{AND}(Z < 1.5, L$11 <= 5), -0.0552*Z^6+0.2175*Z^5-0.2136*Z^4-0.1507*Z^3-0.0856*Z^2+1.0145*Z+0, \text{IF}(\text{AND}(Z > 1.5, L$11 >= 5), 0.0198*Z^5-0.3261*Z^4+2.0962*Z^3-6.4768*Z^2+9.1767*Z-4.0034, \text{IF}(\text{AND}(Z < 1.5, L$11 = 4), -0.0552*Z^6+0.2175*Z^5-0.2136*Z^4-0.1507*Z^3-0.0856*Z^2+1.0145*Z+0, \text{IF}(\text{AND}(Z > 1.5, L$11 = 4), -0.0376*Z^4+0.4664*Z^3-2.0326*Z^2+3.3284*Z-1.041, \text{IF}(L$11 = 2, -2.3599*Z^7+13.8516*Z^6-31.3601*Z^5+35.1888*Z^4-20.9114*Z^3+5.6052*Z^2+0.4698*Z, \text{IF}(L$11 = 3, -0.4268*Z^9+4.7435*Z^8-21.4445*Z^7+50.9252*Z^6-68.5924*Z^5+53.0533*Z^4-23.0229*Z^3+4.8467*Z^2+0.6119*Z, "--"))))))
\]

\[
B_m \text{ or } O12 = \text{IF}(\text{AND}(Z < 1.5, O$11 <= 5), -0.2279*Z^6+0.8659*Z^5-1.1393*Z^4+0.582*Z^3-0.2603*Z^2+0.0316*Z+1, \text{IF}(\text{AND}(Z > 1.5, O$11 >= 5), 0.0039*Z^5-0.0705*Z^4+0.4808*Z^3-1.4355*Z^2+1.4616*Z+0.3803, \text{IF}(\text{AND}(Z < 1.5, O$11 = 4), 0.0229*Z^6+0.8659*Z^5-1.1393*Z^4+0.582*Z^3-0.2603*Z^2+0.0316*Z+1, \text{IF}(\text{AND}(Z > 1.5, O$11 = 4), -0.0603*Z^7+0.573*Z^6-1.9801*Z^5+3.4467*Z^4-3.1852*Z^3+1.0065*Z^2-0.1385*Z+1, \text{IF}(\text{AND}(Z < 2, O$11 = 3), 0.7054*Z^7-4.126*Z^6+9.2089*Z^5-9.8476*Z^4+5.1559*Z^3-1.3884*Z^2+0.1268*Z+1, \text{IF}(\text{AND}(Z >= 2, O$11 = 3), 0.0893*Z^3-0.382*Z^2-0.0873*Z+1.288, "--"))))))
\]

The relatively high level of complexity of the above formulas should be noted. These formulas are automatically filled in the Excel sheet by a simple automated copy/paste process during the run of the “Fill them up” button or macro. Figure 4 shows the moment values as they appear in the Excel worksheet after the macro is run.

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Table 6: Moment coefficients $A_m$ and $B_m$ (Matlock and Reese [7], Das [3])

<table>
<thead>
<tr>
<th>$Z = z/T$</th>
<th>$Z_{\text{max}} = L/T \geq 5$</th>
<th>$Z_{\text{max}} = L/T = 4$</th>
<th>$Z_{\text{max}} = L/T = 3$</th>
<th>$Z_{\text{max}} = L/T = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_m$</td>
<td>$B_m$</td>
<td>$A_m$</td>
<td>$B_m$</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.2</td>
<td>0.198</td>
<td>0.999</td>
<td>0.198</td>
<td>0.999</td>
</tr>
<tr>
<td>0.4</td>
<td>0.379</td>
<td>0.987</td>
<td>0.379</td>
<td>0.987</td>
</tr>
<tr>
<td>0.6</td>
<td>0.532</td>
<td>0.96</td>
<td>0.532</td>
<td>0.96</td>
</tr>
<tr>
<td>0.8</td>
<td>0.649</td>
<td>0.914</td>
<td>0.649</td>
<td>0.914</td>
</tr>
<tr>
<td>1</td>
<td>0.727</td>
<td>0.852</td>
<td>0.727</td>
<td>0.852</td>
</tr>
<tr>
<td>1.5</td>
<td>0.763</td>
<td>0.638</td>
<td>0.762</td>
<td>0.638</td>
</tr>
<tr>
<td>2</td>
<td>0.628</td>
<td>0.404</td>
<td>0.615</td>
<td>0.404</td>
</tr>
<tr>
<td>2.5</td>
<td>0.405</td>
<td>0.2</td>
<td>0.395</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.225</td>
<td>0.06</td>
<td>0.198</td>
<td>0.073</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-0.042</td>
<td>-0.025</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>-0.033</td>
<td>-0.026</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Another Visual BASIC macro and button named "Draw Moment diagram" is then used to draw the moment diagram along the pile length. This macro also works by re-running previously recorded actions including the insertion of a new chart and its formatting. The generated plot of the moments with respect to depth is shown in Figure 5.

To illustrate how the spreadsheet was used in class and/or assignments, the question would be posed as to likely effect of having say a “stubbier” or thicker pile, say 1m in diameter. The results generated instantaneously, and shown in Figure 6, demonstrate that increasing the stiffness of the pile, increased the distributed moments in it.

5 Hybrid analytical method (HAM)

In an effort to expose students to additional available methods for the analysis of piles under seismic loading, a hybrid approach was formulated which combines the solutions provided by Matlock and Reese [8], and Davisson and Gill [4]. The methods were investigated and their respective strengths and limitations identified. The hybrid approach allows the solution of more complex problems than the LCM such those involving layered soils. It consists primarily of combining analytical analysis procedures which focus on various elements of the problem at hand. As such, the process presented narrows the margins of error and provides the essential data for calculating bending moments, shearing forces and deflections along the pile at the working loads. The fact remains that some factors, such as the non-homogeneity of most of the natural soil deposits and the disturbance to the soil caused by installing piles cannot be reproduced.

The educational benefit sought from this approach is to present the student with a...
### Seismic Design of Pile Foundations

#### Figure 4: The calculation of the moment $M_z$ at a depth equal to $z$

<table>
<thead>
<tr>
<th>Step</th>
<th>$z$</th>
<th>$M_z$ (MN.m)</th>
<th>$A_m$</th>
<th>$B_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.323</td>
<td>0.312</td>
<td>0.312</td>
<td>0.993</td>
</tr>
<tr>
<td>1.5</td>
<td>0.647</td>
<td>0.963</td>
<td>0.963</td>
<td>0.951</td>
</tr>
<tr>
<td>2.25</td>
<td>0.970</td>
<td>0.718</td>
<td>0.718</td>
<td>0.692</td>
</tr>
<tr>
<td>3</td>
<td>1.294</td>
<td>0.774</td>
<td>0.774</td>
<td>0.744</td>
</tr>
<tr>
<td>3.75</td>
<td>1.617</td>
<td>0.753</td>
<td>0.753</td>
<td>0.584</td>
</tr>
<tr>
<td>4.5</td>
<td>1.940</td>
<td>0.653</td>
<td>0.653</td>
<td>0.432</td>
</tr>
<tr>
<td>5.75</td>
<td>2.264</td>
<td>0.510</td>
<td>0.510</td>
<td>0.291</td>
</tr>
<tr>
<td>6</td>
<td>2.587</td>
<td>0.371</td>
<td>0.371</td>
<td>0.173</td>
</tr>
<tr>
<td>6.75</td>
<td>2.911</td>
<td>0.257</td>
<td>0.257</td>
<td>0.084</td>
</tr>
<tr>
<td>7.5</td>
<td>3.234</td>
<td>0.169</td>
<td>0.169</td>
<td>0.124</td>
</tr>
<tr>
<td>8.25</td>
<td>3.556</td>
<td>0.102</td>
<td>0.102</td>
<td>-0.010</td>
</tr>
<tr>
<td>9</td>
<td>3.878</td>
<td>0.044</td>
<td>0.044</td>
<td>-0.024</td>
</tr>
<tr>
<td>9.75</td>
<td>4.204</td>
<td>-0.005</td>
<td>-0.005</td>
<td>-0.022</td>
</tr>
<tr>
<td>10.5</td>
<td>4.526</td>
<td>-0.033</td>
<td>-0.033</td>
<td>-0.010</td>
</tr>
<tr>
<td>11.25</td>
<td>4.851</td>
<td>-0.033</td>
<td>-0.033</td>
<td>-0.012</td>
</tr>
<tr>
<td>12</td>
<td>5.175</td>
<td>0.000</td>
<td>0.153</td>
<td>0.000</td>
</tr>
<tr>
<td>12.75</td>
<td>5.498</td>
<td>0.000</td>
<td>0.546</td>
<td>0.000</td>
</tr>
<tr>
<td>13.5</td>
<td>5.821</td>
<td>0.000</td>
<td>1.337</td>
<td>0.000</td>
</tr>
<tr>
<td>14.25</td>
<td>6.145</td>
<td>0.000</td>
<td>2.727</td>
<td>0.000</td>
</tr>
<tr>
<td>15</td>
<td>6.468</td>
<td>0.000</td>
<td>5.013</td>
<td>0.000</td>
</tr>
</tbody>
</table>

#### Figure 5: An Excel chart showing the moment diagram in a pile along its length

Moment diagram along the pile length

---

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number additional analysis methods in a clear and unifying context, rather than piece-meal.

The format of the Excel worksheet for the “hybrid analytical method” is shown below. The definition of each parameter is provided in the following paragraphs.

5.1 Stiffness factors and pile behavior

The first step in the process is to determine whether the pile will behave as a “short rigid unit” or as an “infinitely long flexible member”. This is done by calculating the stiffness factor for the particular combination of pile and soil. The stiffness factors are governed by the stiffness \((EI)\) value of the pile and the compressibility of the soil. The latter is expressed in terms of a “soil modulus”, which is not constant for any soil type but depends on the width of the pile \(B\) and the depth of the particular soil being considered.

In the case of stiff over-consolidated clays, the soil modulus is generally assumed to be constant with depth. The stiffness factor, \(R\), in units of length, is given by eq (9).

\[
stiffness\ factor, \ R = \frac{4}{\kappa} \sqrt{\frac{EI}{k}} \quad (9)
\]

Vesic proposed in 1961 eq (10) to estimate the value of \(k\) a measure of the relative soil/pile stiffness (Das [3], Poulos and Davis [11]).

\[
k = 0.65 \sqrt{\frac{E_p I_p}{E_s I_p}} \frac{E_s}{1 - \mu_s^2} \quad (10)
\]
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Table 7: Representative Values of $n_h$ (Das [3])

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$n_h$ (kN/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Dry or moist sand</td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td>1800 – 2200</td>
</tr>
<tr>
<td>Medium</td>
<td>5500 – 7000</td>
</tr>
<tr>
<td>Dense</td>
<td>15000 – 18000</td>
</tr>
<tr>
<td>Submerged sand</td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td>1000 – 1400</td>
</tr>
<tr>
<td>Medium</td>
<td>3500 – 4500</td>
</tr>
<tr>
<td>Dense</td>
<td>9000 – 12000</td>
</tr>
</tbody>
</table>

In eq (10), $E_s$ is the Young’s modulus of soil, which is 1500 to 2000$S_u$ for heavily over-consolidated clays, and 750 to 1200$S_u$ for lightly over-consolidated clay (Bowles [1]). $S_u$ denotes the undrained shear strength of soil, $D$ the pile width (or diameter), while $\mu_s$ represents Poisson’s ratio of the soil, which vary between 0.3 and 0.5. For most normally consolidated clays and granular soils, the soil modulus is assumed to increase linearly with depth. The stiffness factor is in this case symbolized by the letter $T$ and calculated as follows:

$$\text{stiffness factor}, T = \sqrt{\frac{EI}{n_h}} \quad (11)$$

In the Excel worksheet, the student only has to enter the soil type and the correct formula is identified and used to calculate the appropriate stiffness factor ($R$ or $T$).

Typical values of the coefficient of modulus variation $n_h$ are shown in Table 7. For soft normally consolidated clays, $n_h = 3500$ to 14000 kN m$^{-3}$. For soft organic silt, $n_h = 1500$ kN m$^{-3}$.
If the soil is layered and we have different values of $n_h$ at different depths, the constant value of $n_h$ that must be used is the result of a linear regression between $n_h \cdot z$ and $z$. For this purpose, a new macro was created within the proposed spreadsheet tool and assigned to a button named “estimate $n_h$” (Figure 7). This macro is as follows:

```vba
Sub linearregressiontogethn()
    Dim t As Integer
    t = 0
    Do
        If Application.WorksheetFunction.IsNumber(Cells(t + 6, 4).Value) Then
            t = t + 1
            Cells(t + 6, 6) = Cells(t + 6, 5) * Cells(t + 6, 4)
            Cells(t + 6, 6).Interior.ColorIndex = 56
        End If
    Loop Until Cells(t + 7, 4) = 0
    Cells(7, 2) = Application.WorksheetFunction.LinEst(Range(Cells(6, 6), Cells(t + 6, 6)), Range(Cells(6, 4), Cells(t + 6, 4)), False, False)
End Sub
```

The idea is that, when the user enters the values of $n_h$ at the different depths (e.g.: cells D7:E18), and runs the macro, a worksheet function available to Visual BASIC called “LinEst” is used to get the straight line that best fits the data. The method used to get this line is the "least squares" method. An array that describes the line is then returned (value of cell B7 in our case).

Having calculated the stiffness factor ($R$ or $T$), the criteria for behavior as a short rigid pile or as a long elastic pile can then be related to the embedded length $L$ as follows:

- $L \leq 2 \times$ stiffness factor \( \Rightarrow \) Rigid
- $L \geq 4 \times$ stiffness factor \( \Rightarrow \) Elastic

### 5.2 Elastic analysis of laterally loaded vertical piles

Matlock and Reese [8] established a series of curves for normally consolidated clay and cohesionless soils for which the elastic modulus of the soil $E_s$ is assumed to increase from zero at the ground surface in direct proportion to the depth. These curves represent the deformed shape of the pile and the corresponding bending moments, (shearing forces and soil reactions) using special coefficients. These coefficients are related to a depth coefficient $Z$ for various values of $Z_{\text{max}}$, where $Z$ is equal to the depth $z$ at any point divided by $T$ and $Z_{\text{max}} = L/T$.

For a linearly increasing soil modulus and a free pile head (subject to lateral load and moment), the deflections and moments in the piles are respectively obtained from equations (12) and (13).

$$y_A + y_B = \frac{A_y HT^3}{E_p J_p} + \frac{B_y M_i T^2}{E_p J_p}$$  \hspace{1cm} (12)
Table 8: Equations for the calculation of bending moments and deflection in free head piles carrying lateral load or moment at their heads in soils of constant modulus

<table>
<thead>
<tr>
<th>Load on pile head</th>
<th>For free-headed pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>M</td>
</tr>
<tr>
<td>Moment</td>
<td>M</td>
</tr>
<tr>
<td>Horizontal load</td>
<td>H</td>
</tr>
<tr>
<td>Horizontal load</td>
<td>H</td>
</tr>
</tbody>
</table>

\[
M_A + M_B = A_m HT + B_m M_t \tag{13}
\]

The \( A \) and \( B \) coefficients are obtained from Figures 8 and 9.

For a linearly increasing soil modulus and a fixed pile head (subject to lateral load), eqs (14) and (15) are respectively used for calculating the deflections and bending moments.

\[
y_F = \frac{F_m HT^3}{E_p I_p} \tag{14}
\]

\[
M_F = F_m HT \tag{15}
\]

The \( F \) coefficients are obtained from Figure 10.

Davisson and Gill [4] have analyzed the case of elastic piles in an elastic soil with constant modulus. The bending moments and deflections are related to the stiffness coefficient \( R \) using the equations of Table 8.

Note that the effect of fixity at the pile head can be allowed for by plotting the deflected shape of the pile from the algebraic sum of the deflections and then applying a moment to the head which results in zero slope for complete fixity, or the required angle of slope for a given degree of fixity.

The dimensionless depth coefficient \( Z \) used in the curves is equal to \( z/R \) and \( Z_{\text{max}} = L/R \). These curves are obtained for free-head piles carrying a moment at the pile head and zero lateral loads or, with zero moment at the pile head and carrying a lateral load. The length \( L \) must be greater than \( 2R \). The needed coefficients are obtained from Figures 11 and 12.
Figure 8: Coefficients for laterally loaded free-headed piles in soil with linearly increasing modulus (after Matlock and Reese [8]): and $A_y$ and $A_m$
Figure 9: Coefficients for piles with moment at free head in soil with linearly increasing modulus (after Matlock and Reese [8]): $B_y$ and $B_m$
Figure 10: Coefficients for fixed headed piles with lateral load in soil with linearly increasing modulus (after Matlock and Reese, 1961 [8]): $F_y$ and $F_m$
Figure 11: Coefficients for free headed piles carrying a moment at pile head in soil of constant modulus (after Davisson and Gill, 1963 [4]): $Y_m$ and $M_m$
Figure 12: Coefficients for free headed piles carrying a lateral load at pile head in soil of constant modulus (after Davisson and Gill [4]): $Y_h$ and $M_h$
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Table 1

<table>
<thead>
<tr>
<th>J</th>
<th>K</th>
<th>M</th>
<th>P</th>
<th>R</th>
<th>T</th>
<th>W</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill them up</td>
<td>Moment in a section along the pile length</td>
<td>Deflection of a section along the pile length</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Am</td>
<td>Em</td>
<td>Zmax</td>
<td>Zmax</td>
<td>Mz</td>
<td>Zmax</td>
<td>Zmax</td>
<td>yZ</td>
</tr>
</tbody>
</table>

Figure 13: Typical output of the calculation of the deflection (yZ) and the moment (Mz) at a depth equal to (z) for a free head pile and a linearly increasing soil modulus

In order to produce a “useful” form of the Hybrid Method which facilitates the process of teaching the method and learning it, all the above curves had to be translated into equations to be used in Excel. Particular points on the curves were therefore chosen and then poly-fitted using MATLAB. The generated equations were then inserted in the Excel worksheets to obtain the moment distribution and the deflection at each section along the pile length.

Figure 13 presents the results of the moment and deflection values for the example problem as they appear in the Excel worksheet after running the “Fill them up” button or macro. Another Visual BASIC macro and button named "Draw Moment and deflection diagrams" automate the drawing of the moment and deflection diagrams along the pile length. The plot of the moment diagram with respect to the pile length is Figure 14. The results obtained using the CLM for the same example are also presented in the same figure for comparison. The two methods appear to yield significantly close values for the moments along the pile length. In essence this validates the developed tool.

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Figure 14: An Excel chart showing the moment diagram in a pile along its length

6 Summary and conclusions: academic use and value

In applied engineering, and particularly in the area of geotechnical engineering the teacher and students are presented with a number of problems where the response and behavior of the element of interest is complex and very difficult to visualize and explain. In essence, most problems involving responses conditioned by soil-structure interaction, fall under that category. Traditionally, the educational process and learning outcomes associated with these problems have left a lot to be desired. In our experience teaching deep foundations and the response of piles, the task of tackling the response of piles to lateral loads and moments has been the most challenging and frustrating. The available methods, the number of parameters involved and the empiricism embedded in many of the approaches compounded the difficulties. In response to the above, a new approach was sought, relying primarily on incorporating the power and functionality of spreadsheets. The basic idea was to facilitate the teaching process by providing the teacher with a method based on real-time demonstration of concepts and which also empowered students to explore the effect of various parameters and variables on the response and behavior of the piles.

In this paper, the process of development and use of a spreadsheet-based tool for the analysis of piles under seismic/lateral loading was documented. The significance of the work presented lies in the reduction of the presentation, comprehension and teaching of highly complex soil-structure interaction problem to an Excel environment where various aspects of the analysis can be easily explored, and their importance and effects on overall pile behavior immediately assessed and plotted. The problem of the seismic design of
Seismic Design of Pile Foundations

deep foundations is only one of a number of soil-structure interaction problems where the application of the analytical and graphing power of Excel could result in an easy-to-use-and-demonstrate product which enhances the process of learning and teaching.

We have already introduced the approach described in this paper to a class on geotechnical earthquake engineering at the American University of Beirut in the Fall 2003 semester. The number of students registered for the course was 18 (10 graduate and 8 undergraduate students). The response to the use of the spreadsheets approach was very positive, in the form of interested and interesting questions and feedback, students taking the spreadsheets to generate parametric studies and proposing improvements to the spreadsheets in form and content. This was from the teaching perspective a marked difference to the response we had prior to the introduction of this approach in the course in question in previous semesters when a traditional didactic approach was the norm.

We believe that the most clearly identifiable improvement allowed by the spreadsheet approach was the adoption of “what if?” scenarios to illustrate the effect and importance of certain parameters such as soil types, pile characteristics, levels and type of loading, etc. in real-class-time. The end of semester evaluation comments from some students, both graduate and undergraduate, included that same point.

In summary, to date our experience with using the spreadsheet tool in teaching the fundamentals of pile-soil interaction under seismic loading to classes of seniors and graduate students in civil engineering has been rewarding. The student response to the exercise presented in this paper has prompted us to explore the adoption of similar approaches for other civil engineering/geotechnical/structural applications.

References


