A BI-OBJECTIVE SIMULATION-OPTIMIZATION APPROACH FOR SOLVING THE DYNAMIC VEHICLES ROUTING PROBLEM IN BICYCLE SHARING SYSTEMS

Ahmed A. Kadri
ECAM-EPMI Cergy-Pontoise
LCOMS, Université de Lorraine
Ahmed-adeloumene.kadri@univ-lorraine.fr

Karim Labadi
ECAM-EPMI, ECS-Lab
13, Bd de l’Hautel, Cergy-Pontoise
k.labadi@ecam-epmi.fr

Imed Kacem
LCOMS
Université de Lorraine
Imed.kacem@univ-lorraine.fr

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ABSTRACT

Bicycle Sharing Systems represent an ecological urban mode of transportation allowing people to rent a bicycle at one of many automatic rental stations scattered around the city. In order to meet the users' demands, bicycle sharing systems must be rebalanced regularly. The problem is to design (near-) optimal vehicles routes to visit and rebalance the stations so as to minimize the number of users who try to collect bikes from empty stations or to deliver bikes to full stations. This paper deals with the vehicles routing problem in the dynamic case by extending our previous work based on stochastic Petri net model coupled with a genetic algorithm. We consider the minimization of the uncomfortable situations overall network stations and the total operational cost simultaneously. The proposed simulation-optimization method is tested on a large set of instances and then applied to Cristolob, a real self-service bicycle system of Creteil city, France.

INTRODUCTION

Bike Sharing Systems (BSS) are a very popular means providing bicycles to users in a cheap and self service way. They are one of the solutions to face many public transportation problems, including traffic congestion, air pollution, global oil prices, and global warming. The systems can be described as a set of stations scattered around a city usually located 300 meters apart, and each one is equipped with a terminal and a limited bicycle stands. This systems are very useful for short travels around a city, a user can pick a bicycle at one of any station and return it to the same or any other station, provided that there is an empty locking berth. Although their apparent success in the world, the exploitation of such transport systems implies crucial challenges and one of them is to ensure users that they will be able to find a bicycle or to park it at each station all the day long. Indeed, some stations have more demand than others, especially during peak hours. Bicycles also tend to collect in stations located in the city centers and stay there. In these situations, if no action is taken by the service provider they rapidly fill or empty, thus preventing other users from collecting or delivering bicycle. At an operational level, in order to be able to meet users demands with a reasonable standard of quality, vehicles are used to balance the bicycles among the stations. The objective is to minimize the number of users who cannot be served, i.e., the number of users who try to take a bike from an empty station or to return it to a full station. In the practice, the rebalancing operation can be carried out in two different modes (Raviv et al. 2011). The first one is static, the bicycle redistribution operation can be carried out during the night when the usage rate of the BSS system is very low and the bicycle repositioning is performed based on the status of the system at that time and the demand forecast for the next day. This problem has been recently addressed in some works (Benchimol et al. 2011; Chemla et al. 2012; Kadri et al. 2014; Kadri et al. 2015; Raviv et al. 2013; Shu et al. 2010; Forma et al. 2010). The second mode is dynamic, the bicycle redistribution operation can be carried out during the day when the usage rate of the BSS is significant and bicycle repositioning is performed based on the current state of the station (Kadri et al. 2013; Kadri et al. 2014; Contardo et al. 2012; Sayarshad et al. 2012).

In our previous works (Labadi et al. 2010; Labadi et al. 2012), we developed a stochastic Petri nets based approach for modelling, control and performance analysis of BSS considered as discrete event systems. However, such models must be parameterized to provide a good performance of such systems. Therefore, we proposed in (Kadri et al. 2013; Kadri et al. 2014) an optimization approach considering these models with the aim of maximizing the QoS.

This paper deals with the vehicles routing problem in the dynamic mode and takes into account both the users satisfaction and the total operational cost. This work can be considered as an extension of our previous work (Kadri et al. 2013; Kadri et al. 2014), where we developed an integrated stochastic Petri net and genetic algorithm approach suitable for performance modeling and optimization for control and balancing purposes of such complex dynamical systems. To the best of our knowledge, except our works no other works has been undertaken on BSS modeling and performance optimization from a stochastic and discrete-event point of view. In (Kadri et al. 2013; Kadri et al. 2014), the objective was to optimize the quality of service in BSS by investigating the impact of the regulation thresholds on users satisfaction.
and by considering a fixed route for the regulation vehicles. In this work, we consider a known intervals of thresholds for each station of the system. In practice, they can be defined according to statistical analysis of data provided from a BSS survey during a given period of normal functioning.

The remainder of this paper is as follow: In section 2 we present a mathematical formulation for the problem and we define the objective and the constraints. We describe in section 3 our BSS Petri net model presented in our last works (Labadi et al. 2010; Labadi et al. 2012; Kadri et al. 2014). In section 4, we present an optimization approach based on coupling the Petri net model and a genetic algorithm. In section 5, we present the experimental results and discuss the efficiency of our approach. Finally, we conclude the paper with some discussions and perspectives.

MODEL FORMULATION

The problem is to design a dynamic routes for vehicles visiting the stations in order to rebalance them, so as to minimize the number of users who try to collect bikes from empty stations or to deliver bikes to full stations. Hence, our bi-objective function (1) includes a term which represents the users' satisfaction defined as a sum of the critical times (empty and full stations), as well as a term representing operating cost which is proportional to the total distance traveled by the repositioning vehicles. The sum of the two components represents the objective to be minimized.

In this work, we consider only a single vehicle routing problem, since the regulation is done by area in real situations. We assume the existence of a depot as the starting point of each vehicle’s route. However, we note that the depot may be viewed as a regular station, except that it typically has a relatively large capacity, large initial inventory and no demand. The above decisions are subject to capacity constraints of the vehicles, the stations and the depot, as well as time constraints concerning the total traveling, loading and unloading times. The latter two components are assumed to be linear with the number of bicycles loaded/unloaded.

Additional modeling choices are associated with the permissible actions of the fleet of vehicles performing the repositioning operation, in particular, limitations on the allowed set of routes which the goal is to visit each station one and only one time in a tour.

Before presenting the mathematical formulation of our problem, we introduce first the following notations, variables, and parameters:

- \( N \): A set of nodes including the network stations, indexed by \( i = 1, \ldots, n \).
- \( N_0 \): A set of nodes, including the network stations and the depot (denoted by \( i = 0 \), \( i = 0, \ldots, n \)).
- \( E_i \): Current number of bicycles at the station \( i \) before the repositioning operation.
- \( R_i^- \): Minimal level of the required number of bicycles at a station \( i \).
- \( R_i^+ \): Maximal level of the required number of bicycles at station \( i \).
- \( d_{i,j} \): Vehicles traveling distance from a station \( i \) to a station \( j \).
- \( t_i \): Arrival time of a vehicle to the station \( i \), \( i = 1, \ldots, n \).
- \( \tau_i \): Represents the gap between \( E_i \) and \( R_i^- \), where:
  \( \tau_i = R_i^- - E_i \) if \( E_i < R_i^- \).
- \( \tau_i^+ \): Represents the gap between \( E_i \) and \( R_i^+ \), where:
  \( \tau_i^+ = E_i - R_i^+ \) if \( E_i > R_i^+ \).
- \( C_v \): Capacity of redistribution vehicle.
- \( D_{v_j} \): Number of available bicycles in the vehicle at station \( i \).
- \( p_{t_i} \): Parking time of the vehicle at a station \( i \).
- \( t_i \): Time required to remove a bicycle from a station and load it to the vehicle.
- \( t_u \): Time required to unload a bicycle from the vehicle to a station.
- \( T_{(M = 0)} \): Total time of simulation.
- \( M_i = 0 \): Represents an unavailability event in a station \( i \).
- \( M_i = C_i \): Represents a saturation event in a station \( i \).
- \( T_{(M = 0)} \): Total time of Unavailability of bicycles at station \( i \) during the simulation (Empty station).
- \( T_{(M = C_i)} \): Total time of saturation event at station \( i \) during the simulation (Full station).
- \( \Delta T \): Event duration.
- \( x_{i,j} \): Binary variable which equal to 1 if a vehicle travels from \( i \) to \( j \), 0 otherwise.

Based on the notations, the following mathematical model can be formulated. First, we introduce the objective function that contains two components: first, the sum of the remaining times in critical situations (empty, full) over all stations \( S_i \in N \), and second the total travel distance for the redistribution vehicle:

\[
\begin{align*}
\text{Minimize} & \quad a_1 \sum_{i = 1}^{T_{(M = 0)}} (T_{(M = 0)}) + a_2 \sum_{i = 0}^{N} \sum_{j = 0}^{N} d_{i,j} x_{i,j} \\
\text{S.C.} & \sum_{j = 1}^{N} x_{i,j} = 1 \\
& \sum_{j = 0}^{N} x_{i,j} = 1 \\
& t_u = 0 \\
& t_i \geq t_i + x_{i,j} d_{i,j} + t_i \tau_i^- + t_i \tau_i^+ + p_{t_i} + (x_{i,j} - 1) \ast M_i \\
& \forall i \neq j, j \in N, i \in N \\
& \sum_{i = 0}^{N} x_{i,j} = 1 \\
& \forall j \in N_0, i \neq j \\
& \sum_{i = 0}^{N} x_{i,j} = 1 \\
& \forall j \in N_0, i \neq j \\
& \tau_i^- \leq C_i - D_{v_j} \\
& \forall i \in N \\
& D_{v_j} \geq \tau_i^- \\
& \forall i \in N \\
& T_{(M = 0)} = \sum_{i = 0}^{T_{(M = 0)}} (M_i = 0) \ast \Delta T \\
& \forall i \in N \\
& T_{(M = C_i)} = \sum_{i = 0}^{T_{(M = C_i)}} (M_i = C_i) \ast \Delta T \\
& \forall i \in N
\end{align*}
\]
\[ x_{ij} \in \{0,1\} \quad (12) \]
\[ \omega_1, \omega_2 > 0 \quad (13) \]

The equation (1) minimizes the weighted objective consisting of the sum of the remaining times of stations in the critical situations (empty and full) and the total distance traveled by the vehicle. The constraint (2) and (3) ensures the start and the end of a tour at the depot. Constraint (4) represents the departure time of the vehicle from the depot, and constraints (5) insure the minimal time required for the displacement of the vehicle from a station \( i \) towards a station \( j \) including parking time when arriving to destination and unloading times. The latter two components are assumed to be linear in the number of bicycles picked/delivered. Constraints (6) (resp.,(7)) are vehicle flow-conservation equations (a station can be visited only one time by a vehicle, resp., only one vehicle can exit a station). Constraints (8) (resp.,(9)) are vehicle capacity constraints (available places in vehicle to pick up bicycles from a station, resp., available bicycle to delivery to a station). Constraints (10) (resp.,(11)) represent the remaining times of the stations in critical situations (empty stations, resp., full stations). Finally, (12) are binary constraints. The weighting values of the objective function are linked to the bike rental pricing and the vehicles cost travel measured by the total distance. This weights can be calculated using the Pareto techniques.

**PETRI NET MODEL FOR BSS**

As introduced, we already proposed a Petri net approach dedicated for BSS modeling and analysis for control purposes. We consider a BSS with \( n \) stations noted by \( S = \{ S_1, S_2, \ldots, S_n \} \). Each station \( (S) \) is equipped with \( C_i \) bicycle stands (the capacity of a station \( S_i \)). The system requires a constant control which consists in transporting bicycles from stations having excess of bicycles to stations that may run out of bicycles soon. In the general way, the main objective of the control system, performed by using redistribution vehicles, is to maintain a bicycle safety level at each station \( S_i \) to ensure the availability of a minimal number of bicycles for pick up denoted \( (R_i^-) \), thus the availability of \( (C_i - R_i^-) \) attachment points for return bicycle at each station.

![Figure 1: PN-GA optimization approach](image)

The Petri Net model consists of three subnets (modules) representing three different functions indicated as follows: (a) the “station control” subnet; (b) the “bicycle flows” subnet; and (c) the “redistribution circuit” subnet. The main function of each subnet is described as follows:

- A “bicycle flows” subnet represents the bicycle traffic flows between the different stations of the network.
- A “station control” subnet represents the control function of the stations to ensure bicycles are available for pick up and vacant berths available for bicycle return at every station \( S_i \).
- A “redistribution circuit” subnet represents the path (circuit) to be followed by the redistribution vehicle in order to visit the different stations of the network.

More details on the dynamic of the developed modeling approach can be found in our previous papers (Labadi [12]-[13]).

**PN-GA OPTIMISATION APPROACH**

Motivated by the randomness of BSS and the NP hardness of the studied problem, we developed an approach based on the integration of the Petri net model for BSS and a genetic algorithm in order to solve the vehicles routing problem. Although Petri nets itself is not an optimization tool, it can provide a practical decision making tool, if it is combined with and enhanced by some optimization algorithms (Kadri et al. 2013; Kadri et al. 2014; Nodhi et al. 2012; Sauer and Xie 1993; Sharda and Bnejee 2013, Sadrrieh et al. 2007). In this part of this work, a genetic algorithm is coupled with the Petri net model in order to solve the problem formulated previously. Our discrete event simulation/optimization approach is illustrated by the Figure 1.
Data setting
We tested our approach for a real case study for the BSS (Cristolib) of Creteil city, France (http://www.cristolib.fr). As shown in Figure 3, the network consists of 10 stations with a total of 130 bicycles. Each station is characterized by its capacity and an average regulation threshold ($R'_i$, $R'_j$), as indicated in Table 1. The travel times of vehicles between the different stations including the depot, estimated using Google map application, are shown in Table 2. For this application, the capacity of the vehicle is fixed to 30 bicycles and the times allocated to a pickup or delivery operation are fixed to 10 seconds per bike. The average frequency of the stations, given in Table 3, represents user requests measured during a 10 days of normal functioning (excluding summer season, special events,...). We also specify that the total simulation time for the execution of the PN representing the system is fixed to two weeks. This time is relatively long, but it allows us to obtain a reliable estimation of the average performance of the system during one day’s service.

<table>
<thead>
<tr>
<th>Station</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
<th>$S_6$</th>
<th>$S_7$</th>
<th>$S_8$</th>
<th>$S_9$</th>
<th>$S_{10}$</th>
</tr>
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<tbody>
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<td>Capacity</td>
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<td>22</td>
<td>24</td>
<td>40</td>
<td>18</td>
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<td>Average threshold</td>
<td>13</td>
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<td>12</td>
<td>20</td>
<td>9</td>
<td>12</td>
<td>10</td>
<td>17</td>
<td>11</td>
<td>10</td>
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</tbody>
</table>

Figure 3: Distribution of bike stations in the network (Source: http://www.cristolib.fr)

Table 2: Estimation of displacement delays of a redistribution vehicle (in minutes), source Google map.

<table>
<thead>
<tr>
<th>Station</th>
<th>$D$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
<th>$S_6$</th>
<th>$S_7$</th>
<th>$S_8$</th>
<th>$S_9$</th>
<th>$S_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
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<td>5</td>
<td>8</td>
<td>9</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>$S_1$</td>
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<td>0</td>
<td>3</td>
<td>3</td>
<td>4</td>
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<td>6</td>
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<td>9</td>
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<tr>
<td>$S_2$</td>
<td>8</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>6</td>
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<td>6</td>
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<td>7</td>
<td>11</td>
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<tr>
<td>$S_4$</td>
<td>12</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>4</td>
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<tr>
<td>$S_5$</td>
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<td>6</td>
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<tr>
<td>$S_6$</td>
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<td>6</td>
<td>4</td>
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<td>0</td>
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<td>$S_7$</td>
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<td>$S_8$</td>
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<td>$S_{10}$</td>
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<td>5</td>
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<td>4</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 2: Genetic Operators
As shown in Figure 2(a), the chromosome representing a vehicle route constitutes sequence, the genes contain the assignments of jobs for a vehicle in their execution order, thus a job represents a displacement of a vehicle from a station $i$ to $j$. Different genetic operators are applied as crossovers and mutations under fixed probabilities. After evaluation, each individual has an adjustment level which presents the value of the bi-objective function defined in equation (1) of the mathematical formulation. The evaluation of the objective is done via the simulation of PN-model and the evaluation of simulation results. We note that this step is the most expensive in terms of execution time of our GA.

A REAL APPLICATION CASE

We describe in this section the numerical experiments carried out in order to evaluate our approach. To integrate the Petri net model and the genetic algorithm, we developed in C ++ language a tool for both modeling, simulation, performances evaluation and parameters/decisions optimization of BSS. The experimentations were carried out on an Intel Core i7 2.7 GHz processor and 8 GB of RAM, under Windows 8 environment.
Table 3: Average travel demand matrix, source: statistical average of user requests for a day.

<table>
<thead>
<tr>
<th>Station</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
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<th>$S_9$</th>
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<tbody>
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<td>6</td>
<td>4</td>
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<td>$S_{10}$</td>
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</table>

Algorithm Setting

We note that the variations of GA parameters can impact on the quality of solution. In this application, we set the GA parameters as follows: the initial population size is set to 20, we consider a one point crossover with a probability ($P_{crossover}=0.9$) and we perform one mutation by swapping two elements in chromosomes with a probability ($P_{mutation}=0.1$). The iterations number is limited to 200.

We remember that setting the weights $\alpha_1$ (resp. $\alpha_2$) of the bi-objective function weighting the critical times (resp. total distance) is a pricing cost problem since they are linked to the bike rental pricing and (resp. the operational cost of a vehicle which is measured by the total traveled distance).

One of the advantages of our Petri net model is its associated performance analysis methods which allow an efficient analysis and evaluation of the system behavior during simulation time. In these tables, several performance indices are given for each station such as:

- The average time ($\%$) where a station remains in empty (resp. full) state is calculated by using the equations:

  \[
 \text{Empty rate} \ (S_i) = 100 \times \frac{T_{sim} - \sum (t_i - t_{i+1})_{M(PS_i)=0}}{T_{sim}} \quad (14)
  \]

  \[
  \text{Full rate} \ (S_i) = 100 \times \frac{T_{sim} - \sum (t_i - t_{i+1})_{M(PS_i)=C_i}}{T_{sim}} \quad (15)
  \]

- Average number of bicycles ($S_i$) = $\frac{\sum M(PS_i) \times \tau}{T_{sim}}$ \quad (16)

The average critical time ($\%$) where a station remains in empty or full states (i.e., sum of the equations (14) and (15)). We define also the quality of service $QoS$ for each station as the average service time ($\%$) defined by the equation (17).

The quality of service of the network is then ($\sum QoS$) / $n$.

\[
QoS = \left[ 1 - \left( \frac{T_{sim} - \sum (t_i - t_{i+1})_{M(PS_i)=0}}{T_{sim}} \right) + \left( \frac{T_{sim} - \sum (t_i - t_{i+1})_{M(PS_i)=C_i}}{T_{sim}} \right) \right] \times 100 \quad (17)
\]

Where $T_{sim}$ is the total simulation time; $(t_i - t_{i+1})$ represents the cycle duration where the station $S_i$ is empty (resp. full)

(i.e., $M(PS_i)=0$ resp. $M(PS_i)=C_i$ ($C_i$ represents the capacity of the station $S_i$).

Table 4 : System performance with the best vehicle route.

<table>
<thead>
<tr>
<th>Station</th>
<th>Empty time</th>
<th>Saturation time</th>
<th>Critical time</th>
<th>QoS</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>0</td>
<td>19.44</td>
<td>19.44</td>
<td>80.56</td>
<td>20/26</td>
</tr>
<tr>
<td>$S_2$</td>
<td>0</td>
<td>12.71</td>
<td>12.71</td>
<td>87.29</td>
<td>15/22</td>
</tr>
<tr>
<td>$S_3$</td>
<td>4.96</td>
<td>1.53</td>
<td>6.49</td>
<td>93.51</td>
<td>10/24</td>
</tr>
<tr>
<td>$S_4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>20/40</td>
</tr>
<tr>
<td>$S_5$</td>
<td>2.29</td>
<td>4.32</td>
<td>6.61</td>
<td>93.39</td>
<td>10/18</td>
</tr>
<tr>
<td>$S_6$</td>
<td>1.99</td>
<td>0.89</td>
<td>2.88</td>
<td>97.12</td>
<td>10/24</td>
</tr>
<tr>
<td>$S_7$</td>
<td>12.29</td>
<td>0</td>
<td>12.29</td>
<td>87.71</td>
<td>5/20</td>
</tr>
<tr>
<td>$S_8$</td>
<td>11.20</td>
<td>0</td>
<td>11.2</td>
<td>88.8</td>
<td>8/34</td>
</tr>
<tr>
<td>$S_9$</td>
<td>4.12</td>
<td>0.35</td>
<td>4.47</td>
<td>95.53</td>
<td>8/22</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>0</td>
<td>9.55</td>
<td>9.55</td>
<td>90.45</td>
<td>14/20</td>
</tr>
</tbody>
</table>

Average 3.68 4.87 8.56 91.4

Algorithm convergence and results analysis

Table 4 provides the system performances when the total distance is favored ($\alpha_1=0$; $\alpha_2=1$). Different metrics of performances are given for each station such as empty times, full times, critical times, availability rate of bicycles and finally the $QoS$. The terms that comprise the objective (i.e., critical times and total distance) are assessed separately in table 5. However, the provided solution is not necessary optimal due to the used approach and the NP-hardness of the studied problem, but it can be a good initiative to optimize the objective in such stochastic systems.

To test the impact of the weights associated to the objective function on the quality of solutions, we test some variation of $\alpha_1$ and $\alpha_2$ by favoring one over the other, then we evaluate separately the cost of each term of the objective function (i.e., the total critical times and the total distance). The grounds that the quantity of the first term of the objective function is very smaller than the second term, the weighting of this last must be less than the first term except for cases where the $QoS$ is not favored. For the considered tests, we keep the same input data. The different configurations and results are given in Table 5.

Table 5 : Evaluation of the weightings impact on the bi-objective function.

<table>
<thead>
<tr>
<th>Approach</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>Critical times (% of total service time)</th>
<th>Total distance (vehicle)</th>
<th>Full Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN-GA</td>
<td>1</td>
<td>0</td>
<td>$3.21$</td>
<td>5410</td>
<td>3.21</td>
</tr>
<tr>
<td>PN-GA</td>
<td>1500</td>
<td>1</td>
<td>5.49</td>
<td>4970</td>
<td>13205</td>
</tr>
<tr>
<td>PN-GA</td>
<td>800</td>
<td>1</td>
<td>4.91</td>
<td>4320</td>
<td>8248</td>
</tr>
<tr>
<td>PN-GA</td>
<td>100</td>
<td>10</td>
<td>8.56</td>
<td>$3300$</td>
<td>3300</td>
</tr>
<tr>
<td>PN-GA</td>
<td>0</td>
<td>8.56</td>
<td>$3300$</td>
<td>$3300$</td>
<td></td>
</tr>
<tr>
<td>Dijkstra - PN</td>
<td>-</td>
<td>8.56</td>
<td>$3300$</td>
<td>$3300$</td>
<td></td>
</tr>
</tbody>
</table>

One can observe that the variation of weights can affect the objective, as we can observe it particularly when one of the weighting is set to zero. The weights impact definitely on the decisions related to the vehicles routes and the value of the objective function. Since, setting of weights depend on pricing policies that are linked to profits of each objective.
However, adjusting these settings manually is invalid. Hence, setting the weights becomes an auxiliary NP-hard problem. This can be separately solved by finding a Pareto optimal points for the corresponding bi-objective function (Mostaghim and Teich 2005), which allow us to obtain a meaningful solution that meets the objectives. When solving this problem, the parameters related directly to the objective function must be known, such as the pricing rental of bicycles and the cost of route (for example 2€ for 1 Km).

In this work, we limit ourselves to the presentation of the modeling-optimization approach and we certainly intend to solve the Pareto optimal in future works.

As shown in figure 4, we observe the evolution of the best solution for the different configurations during the iteration number and therefore improving the objective. In our approach, we consider both the users satisfaction measured by the critical times and the operational cost measured by the total traveled distance by a rebalancing vehicle, and considering the stochastic demand of users.

CONCLUSION

Bicycle sharing systems have emerged around the world as a viable urban mobility alternative, being already widely spread. These systems have been quickly evolving in the last decades and currently they are integrated with other existing transportation modes in many cities. This study investigates the vehicles routing problem in bicycle sharing systems considering the dynamic case. We define and formulate a mathematical model taking into account the time required to perform the operation with the aim to minimize both the critical situations through the networks stations as well as the operational cost for the service providers.

Motivated by the randomness of BSS and the NP-hardness of the studied problem, we developed an integrated approach based on coupling between a genetic algorithm and BSS-stochastic Petri Net model in order to find the best possible routes for the redistribution vehicles that minimize the objective. The developed approach integrates the users satisfaction, the quality of service, as well as the operational cost. Our approach was tested for a real problem. According to the best of our knowledge, except our previous work, this approach is the first in literature based on coupling genetic algorithm with Petri nets model for solving the vehicles routing problem in BSS, such an approach, should be ranked among the predictive approaches investigating the operational problems in BSS. Finally, this work addresses both the design and the management of bicycle sharing systems considering the stochastic behaviour of such systems.

In future works, we aim to present some mathematical formulations investigating other operational problems in bicycle sharing systems and thereby the development of efficient metaheuristic methods and/or exact algorithms to solve problems of large sizes.

REFERENCES


**BIOGRAPHIES**

**AHMED ABDELMOUMENE KADRI** received his Engineer degree in Industrial IT from the Université d’Oran (Es Senia), Algeria in 2010 and his Master’s degree in Computer sciences from the Université de Lorraine, France in 2012. He is currently working toward his PhD in computer engineering at the “Laboratoire LCOMS” of the “Université de Lorraine” (Metz, France) and the Graduate School of Electrical Engineering and Industrial Management (ECAM-EMPI), Cergy-Pontoise, France. His research interests include operational research, combinatorial optimization, modeling, performance evaluation and optimization of stochastic systems such in logistic and transportation field.

**KARIM LABADI** is currently an Assistant Professor at the School of Electricity, Production and Industrial Management (EPMI) and member of the ‘Equipe Commande des Systèmes’ (ECS-Lab, EA.3649) at the ‘École Nationale Supérieure d’Electronique et ses Applications’ (ENSEA), Cergy-Pontoise France, since 2006. He obtained his M.S degree in Applied Automation and Informatics from the IRCCyN, ‘École Centrale de Nantes (ECN)’, France in 2002, and PhD in Industrial Engineering and Optimization from the University of Technology of Troyes (UTT), France in 2005. His research interests include Petri nets and their applications to modelling, control and performance optimization of stochastic discrete event systems such as production, transportation and logistic systems.
IMED KACEM is a Full Professor at the University of Lorraine (France) where he created the LCOMS laboratory in 2011. He is the Head of this multidisciplinary laboratory (http://lcoms.univ-lorraine.fr). He is a nominated member at the National Council of the Universities (Computer Science Section CNU27, France), Associate or Guest Editor for several referred journals (such as EJIE, JSSSE (Springer), Computers & Industrial Engineering (Elsevier) ...) and organizer of major international conferences (CoDIT’14, IEEE/CIE’39, IEEE/ICSSSM’06, WAC/ISIAC’06). His scientific activity is in a transversal and interdisciplinary domain: the Operational Research. More precisely, his contributions are related to the design of exact and approximate algorithms with a guaranteed performance for the NP-hard combinatorial problems. He obtained the « 2010 Great Award of Research » from the Universities of Lorraine and the « 3rd Robert Faure Award 2009 » from the French Society of Operational Research and he has regularly the PEDR or the PES Premium (level A) since 2006.