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This classroom note shows how one can use a spreadsheet in developing conceptual understanding of the decimal system of arithmetic through exploring representations of integers in non-decimal base systems. In programming a spreadsheet to execute even simple operations (usually taken for granted) such as the generation of counting numbers or finding the sum of two numbers when the base number is smaller than 10, the need for conceptualization of those operations becomes obvious. In particular, a ready-made spreadsheet provides an effective medium for prospective elementary teachers who, by experiencing and overcoming difficulties in doing a non-traditional arithmetic, become better prepared to teach the conventional place value system to young children. Likewise, prospective high school teachers can experience the use of mathematical concepts, like the logarithmic function, as tools in computing applications when developing such a spreadsheet.

Keywords

spreadsheet, base system, flow chart, face value, place value

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Computing With Spreadsheets In Different Base Systems

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“The ability to shift at will one system to another (e.g., to “translate” from the decimal system into one that is based on five) . . . indicates the existence of a general concept of a system of numeration”
Vygotsky [7, p115]

Abstract

This classroom note shows how one can use a spreadsheet in developing conceptual understanding of the decimal system of arithmetic through exploring representations of integers in non-decimal base systems. In programming a spreadsheet to execute even simple operations (usually taken for granted) such as the generation of counting numbers or finding the sum of two numbers when the base number is smaller than 10, the need for conceptualization of those operations becomes obvious. In particular, a ready-made spreadsheet provides an effective medium for prospective elementary teachers who, by experiencing and overcoming difficulties in doing a non-traditional arithmetic, become better prepared to teach the conventional place value system to young children. Likewise, prospective high school teachers can experience the use of mathematical concepts, like the logarithmic function, as tools in computing applications when developing such a spreadsheet.

Key words: spreadsheet, base system, face value, place value, teacher education,

1 Introduction

This classroom note stems from the author’s work with K-12 teacher candidates towards their mathematical and technological preparation. In this work, several leading documents [3], [4], [5] setting up recommendations and standards for teaching mathematics guided the author. In what follows, the use of a spreadsheet in teaching arithmetic conceptually as well as utilizing the concepts of arithmetic and algebra in spreadsheet programming will be illuminated. In the context of elementary teacher

preparation, a quote from the Conference Board of the Mathematical Sciences [4] recommendations for the mathematical education of teachers ascribing the fundamental importance to “the place value structure of our number system, which implicitly expresses numbers as polynomials in powers of 10 and permits single-digit arithmetic to be easily extended to multi-digit arithmetic” (p. 5) is worthy to be mentioned. A deep understanding of place value by prospective elementary teachers is possible, as Vygotsky [7] suggested in the context of teaching schoolchildren, by “experimenting with shifts from decimal to other numerical systems” (p. 115). Such experiments can even be supported by a physical manipulative. Although Schmittau & Vagliardo [6] expressed their concern regarding the absence of the multi-base blocks in the classroom, something that, in their view, resulted from “the advent of the back to basics movement”, the use of square tiles can replace effectively these blocks. To a certain extent, square tiles are even more operational tools for understanding the mechanism of transition from one base to another allowing for a greater flexibility in the use of hands-on strategies.

Likewise, a spreadsheet can be used as an electronic manipulative to create diagrammatic representations of operations in different bases as shown below in Figure 6 and in more detail elsewhere [1]. Furthermore, regarding the preparation of high school teachers and their students alike to use technology as a means of enhancing one’s knowledge of mathematics, it is a sense of agreement among educators “that spreadsheet work can illuminate important mathematical ideas” [4, p128]. Indeed, as will be shown in this note, a spreadsheet can be used as a computational tool the programming of which and subsequent didactically-appropriate utilization can help develop a deeper understanding and conceptualizing of place value arithmetic through transition from the decimal to non-decimal spreadsheet-based representations of numbers. A special course on the use of spreadsheets in teaching school mathematics like the one developed by the author for prospective high school teachers [2] may be a venue for practicing skills in using concepts of arithmetic and algebra in programming spreadsheet-based operations on numbers in different base systems.

2 Representation of digits in an arbitrary base

Let an n -digit integer N in decimal positional system has the following representation

$$N = a_n \cdot 10^n + a_{n-1} \cdot 10^{n-1} + \dots + a_{n-i} \cdot 10^{n-i} + a_{n-i-1} \cdot 10^{n-i-1} + \dots + a_1 \cdot 10 + a_0,$$

where $0 \leq a_i < 10$ for $0 \leq i < n$ and the leading digit $1 \leq a_n < 10$. Then its digits a_i have the following representation through the function $\text{INT}(x)$ which returns the greatest integer smaller or equal to x :

$$a_i = \text{INT}\left(\frac{N}{10^i}\right) - 10 \cdot \text{INT}\left(\frac{N}{10^{i+1}}\right), 0 \leq i \leq n \quad (1)$$

Indeed,

$$\begin{aligned}
 & INT\left(\frac{N}{10^i}\right) - 10 \cdot INT\left(\frac{N}{10^{i+1}}\right) \\
 &= INT\left(\frac{a_n \cdot 10^n + a_{n-1} \cdot 10^{n-1} + \dots + a_i \cdot 10^i + a_{i-1} \cdot 10^{i-1} + \dots + a_0}{10^i}\right) \\
 &- 10 \cdot INT\left(\frac{a_n \cdot 10^n + a_{n-1} \cdot 10^{n-1} + \dots + a_{i+1} \cdot 10^{i+1} + a_i \cdot 10^i + \dots + a_0}{10^{i+1}}\right) \\
 &= a_n \cdot 10^{n-i} + a_{n-1} \cdot 10^{n-i-1} + \dots + a_{i+1} \cdot 10 + a_i \\
 &- (a_n \cdot 10^{n-i} + a_{n-1} \cdot 10^{n-i-1} + \dots + a_{i+1} \cdot 10) = a_i.
 \end{aligned}$$

A similar formula,

$$d_i = INT\left(\frac{N}{B^i}\right) - B \cdot INT\left(\frac{N}{B^{i+1}}\right), 0 \leq i < m, \quad (2)$$

returns a digit d_i of the same number in base B , where $\lfloor x \rfloor$ is the greatest integer smaller than or equal to x , described in the spreadsheet context as $INT(x)$, and $m = \lfloor \log_B N \rfloor + 1$ is the number of digits of N in base B . Indeed, setting

$$N = d_m \cdot B^m + d_{m-1} \cdot B^{m-1} + \dots + d_{i+1} \cdot B^{i+1} + d_i \cdot B^i + \dots + d_1 \cdot B + d_0, \quad (3)$$

as the representation of N in base B through the digits d_i , it follows that

$$\begin{aligned}
 & INT\left(\frac{N}{B^i}\right) - B \cdot INT\left(\frac{N}{B^{i+1}}\right) \\
 &= INT\left(\frac{d_m \cdot B^m + d_{m-1} \cdot B^{m-1} + \dots + d_i \cdot B^i + d_{i-1} \cdot B^{i-1} + \dots + d_0}{B^i}\right) \\
 &- B \cdot INT\left(\frac{d_m \cdot B^m + d_{m-1} \cdot B^{m-1} + \dots + d_{i+1} \cdot B^{i+1} + d_i \cdot B^i + \dots + d_0}{B^{i+1}}\right) \\
 &= d_m \cdot B^{m-i} + d_{m-1} \cdot B^{m-i-1} + \dots + d_{i+1} \cdot B + d_i \\
 &- (d_m \cdot B^{m-i} + d_{m-1} \cdot B^{m-i-1} + \dots + d_{i+1} \cdot B) = d_i.
 \end{aligned}$$

3 Computational separation of digits in base ten

Figure 1 shows how, according to formula (1), the use of the spreadsheet formula

$$\begin{aligned}
 &=IF(A2<=LEN(Number),INT(Number/10^(LEN(Number)-A2)) \\
 &-10*INT(Number/10^(LEN(Number)-A2+1)), "")
 \end{aligned}$$

defined in cell B2 and replicated down column B, enables the computational separation of digits of a number displayed in cell B1 and given the name Number. When a digit's positional rank (displayed in column A) is not greater than the total

number of digits in the number, defined as LEN(Number), this formula yields a corresponding digit, otherwise, it leaves the cell blank.

	A	B
1		123029
2	1	1
3	2	2
4	3	3
5	4	0
6	5	2
7	6	9
8	7	
9	8	
10	9	
11	10	

Figure 1. Computational separation of digits in base 10.

4 Computational separation of digits in a non-decimal base

A similar spreadsheet formula, based on formula (2), can be used to computationally transform a base ten number into a string of digits that represents the number in a non-decimal base. For example, because $47 = 1 \cdot 3^3 + 2 \cdot 3^2 + 0 \cdot 3^1 + 2 \cdot 3^0$, the number 47 in base 3 is the string 1202 and the following formula defined in cell B2 (Figure 2) and replicated down column B enables the separation of digits in this (or any other) string:

=IF(INT(LOG(Number, Base))+1-A2>=0, INT(Number/Base^(INT(LOG(Number, Base))+1-A2))-Base*INT(Number/Base^(INT(LOG(Number, Base))+1-A2+1))," ")

Here the name Number refers to a given number in base ten, and Base is the name given to a base number defined in cell A1.

	A	B
1	3	47
2	1	1
3	2	2
4	3	0
5	4	2
6	5	
7	6	

Figure 2. Transforming a base ten number into a string of digits in a different base

The difference between the two spreadsheet formulas (developed for the spreadsheets in Figure 1 and Figure 2, respectively) is that the latter uses the construction $\text{INT}(\text{LOG}(\text{Number}, \text{Base}))+1$ in place of $\text{LEN}(\text{Number})$. For example, the number of digits in the string 1202 which represents 47 in base 3 results from the equality $\text{INT}(\text{LOG}(47, 3))+1=4$. Alternatively, the formula $\log_b N = \frac{\log_{10} N}{\log_{10} B}$ can be used. Note that

although the digits in the string 1202 show the quantities of the corresponding powers of the base number 3 in the representation of 47, the very number 1202 to be displayed by a spreadsheet results from a linear combination of the powers of ten, $1 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10^1 + 2 \cdot 10^0$.

5 Translating numbers from base ten into other bases

In order to turn a string of digits in base B into a number in this base, one has to use formula (3). Towards this end, the spreadsheet formula

$$=\text{IF}(B2=" ", " ", \text{INT}(10^{(\text{CEILING}(\text{LOG}(B\$1, \text{Base}), 1) - A2)}))$$

is defined in cell C2 and replicated down column C yielding the integer powers of ten. Finally, the formula $=\text{SUMPRODUCT}(B2:B11, C2:C11)$ is defined in cell C1 yielding the representation of base ten number from cell B1 in a base defined in cell A1.

	A	B	C
1	3	47	1202
2	1	1	1000
3	2	2	100
4	3	0	10
5	4	2	1
6	5		
7	6		

Figure 3. Transforming a string of digits into a number.

6 Generating a sequence of consecutive numbers in an arbitrary base

The spreadsheet of Figure 4 is designed to generate consecutive counting numbers in a base system. For example, the 15th counting number in base six has the representation 23 (Figure 4, cell R2), i.e., $23_{base\ 6} = 2 \cdot 6^1 + 3 \cdot 6^0$. The spreadsheet is programmed as follows.

Cell A1: a base number is entered and given the name Base.

In the range A2: A11 consecutive counting numbers are generated (hidden from view). Likewise, these numbers are generated in row 1 beginning cell D1 (hidden from view). Cells A1 and B1 are controlled by sliders, which generate the base number and consecutive counting (base ten) numbers starting from 1, respectively. In columns B and C, beginning from cells B2 and C2, respectively, the face values and place values of a number displayed in cell B1 are generated through the following formulas:

cell B2: =IF(INT(LOG(B\$1, Base))+1-A2>=0,

INT(B\$1/A\$1^(INT(LOG(B\$1,Base))+1-A2))-A\$1*INT(B\$1/A\$1^(INT(LOG(B\$1, Base))+1-A2+1)), " ");

cell C2: =IF(B2=" ", " ", IF(CEILING(LOG(B\$1, Base),1)

= LOG(B\$1, Base),

10^(LOG(B\$1, Base)-A2+1), INT(10^(CEILING(LOG(B\$1, Base),1)-A2)))).

The last two formulas are replicated down columns B and C, respectively; the results of computations are hidden from view also.

In cell C1, through the formula =SUMPRODUCT(B2:B11,C2:C11) the representation of a base ten number displayed in cell B1 in a base displayed in cell A1 is generated (in Figure 4 we have $14_{base\ 10} = 22_{base\ 6}$). Finally, cell D2 is entered with the number 1 and cell E2 with the formula =IF(\$B1=1,"",IF(E1=\$B\$1,\$C\$1,E2)) which is then replicated to the right across row 2. As the slider generates consecutive counting (base 10) numbers in cell B1, the last formula generates their representations in a base displayed in cell A1.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	6	14	22														
2				1	2	3	4	5	10	11	12	13	14	15	20	21	22
3																	
4																	

Figure 4. Generating consecutive counting numbers in a base system.

7 Developing addition table in a base system

The spreadsheet pictured in Figure 5 represents an addition table in a base system. It uses jointly programming techniques already discussed above and new formulas described below. Consecutive counting numbers in base B system are developed by changing a number in cell B2 the representation of which in base B is a number in cell C4 and base B is displayed in cell A4. Then the following formulas =IF(H\$4=""," ",INT(H\$4/(10^(\$F5-1))-10*INT(H\$4/(10^(\$F6-1))))

and

=IF(\$G5=""," ",INT(\$G5/(10^(H\$3-1))-10*INT(\$G5/10^(I\$3-1))))

are defined in (and replicated to) cells H1 (Q2) and D5 (E14), respectively. These formulas generate face values of the numbers appearing in the ranges H4:Q4 and G5:G14, respectively. Finally, the formula

=IF(OR(\$B\$4=1,H\$4="",\$G5="")," ",

IF(H\$1+\$D5>=Base,10*(H\$2+\$E5+1)+H\$1+\$D5-Base, 10*(H\$2+\$E5)+H\$1+\$D5))

is defined in cell H5 and replicated to cell Q14.

	A	B	C	F	G	H	I	J	K	L	M	N	O	P	Q
3															
4		6	10	14											
5					1	2	3	4	5	10	11	12	13	14	15
6					2	3	4	5	10	11	12	13	14	15	20
7					3	4	5	10	11	12	13	14	15	20	21
8					4	5	10	11	12	13	14	15	20	21	22
9					5	10	11	12	13	14	15	20	21	22	23
10					10	11	12	13	14	15	20	21	22	23	24
11					11	12	13	14	15	20	21	22	23	24	25
12					12	13	14	15	20	21	22	23	24	25	30
13					13	14	15	20	21	22	23	24	25	30	31
14					14	15	20	21	22	23	24	25	30	31	32

Figure 5. Addition table in base six.

Note that the formulas used in developing the spreadsheet pictured in Figure 5 are limited to a certain range and base systems. However, the role of a spreadsheet in the context of this classroom note is not to develop a computational tool that works without limitations but rather, in the context of the software to develop skills in conceptualizing the development of number and operations with numbers in a non-decimal base system. The formulas can be extended to accommodate addition of larger numbers in smaller bases.

8 Classroom applications

Spreadsheet environments that can carry out operations in non-decimal bases have used by the author with teacher candidates in several ways. One of the assignments for a mathematics content course for elementary pre-teachers includes the need to partition a base-B number into a sum of two like numbers in all possible ways using diagrams and then describe the partitioning diagrams symbolically. For example, consider the number 13_{base6} which in Figure 5 runs diagonally from cell H12 to cell O5. Using information presented in the spreadsheet, one can describe all the partitions as follows:

$$13 = 12 + 1 = 11 + 2 = 10 + 3 = 5 + 4 = 4 + 5 = 3 + 10 = 2 + 11 = 1 + 12 \quad (4)$$

(here all two-digit numbers are base-6 numbers). Equalities (4) can then be compared with partitioning diagrams also created within a spreadsheet and shown in Figure 6. It should be noted that the spreadsheet of Figure 5 (and like environments that can be constructed by mathematics educators to be used in a teacher education classroom) is recommended to be used by elementary pre-teachers as ready-made computational tools.

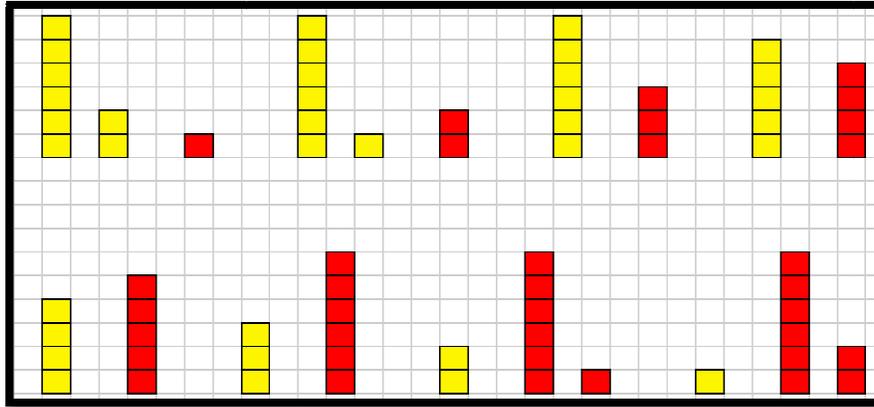


Figure 6. Diagrammatic representation of equalities (4).

As one elementary pre-teacher put it, “When children start doing base-ten arithmetic, it must be as confusing to them as other base systems to me. If I could develop a conceptual understanding of arithmetic in other bases, this technique should also be helpful in teaching students base ten for the first time. When students don’t understand the concept and what is actually going on, math seems mysterious and more confusing. As teachers, it’s important that we use techniques to help students really understand the concept behind what they are doing. By doing this, it will help make students more successful.”

Another elementary pre-teacher went even further by expressing willingness to learn about spreadsheet programming details. In her words (found by the author in a class assessment): “I would like to learn how the tools we used were created.” This remark, echoing a theoretical tradition of learning in the zone of proximal development [8], suggests that knowing about interest of elementary pre-teachers towards mathematical ideas and concepts typically considered “too complex” can motivate one to reconsider the conventional boundaries of mathematics curriculum for this population of teacher candidates.

In a course for prospective high school teachers on the use of spreadsheets in teaching school mathematics [2], one of the assignments includes activities on programming a spreadsheet through which mathematical concepts are used as tools in computing applications. For example, while the logarithmic function is well known as a formal entity at the high school level, its possible applications are not generally known. One such application is shown in section 4 of this note when formalizing the definition of the number of digits in a given base. When computing in base ten, one automatically names a two-digit number 10 after the number 9 is named. How can one teach an artificial intellect, like a spreadsheet, to do that? Programming activities that address this question are very useful from both mathematical and computational perspectives for those preparing to teach high school mathematics.

9 Conclusion

The classroom note offered a few teaching ideas of using a spreadsheet in mathematics teacher education to support a classic recommendation [7] regarding the need to experiment with different base systems for a better understanding the commonly used decimal system. The note also supported a more recent recommendation [4] for prospective teachers of mathematics concerning the need to have a deep conceptual understanding of the place value structure of base-ten system. A limitation of a spreadsheet in carrying out operations in different bases can motivate one to improve algorithms used in this note when looking for an independent project in any teacher education course on the use of technology in mathematical problem solving or in a basic computer science course.

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