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Mass, Measurement, Materials, and Mathematical Modeling: The Nuts and Bolts of Extrapolation

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Mass, Measurement, Materials, and Mathematical Modeling: The Nuts and Bolts of Extrapolation

Abstract
A simple activity is described which is appropriate for any class dealing with measurement**.** It introduces students to the important scientific process of mathematical modeling and online collaboration. Students, working in groups, determine the mass of a bolt indirectly by extrapolation from massing the bolt with one to five nuts on it and determining the equation of the line; the y-intercept being the mass of the bolt. Students gain experience with using a balance, graphing data, and analyzing results using algebraic skills. They calculate percent error after measuring the bolt's mass directly and can compare this with the error limits from the least squares fit. Groups enter data into a web-based form and the data is examined by the class using Google Docs in a collaborative manner. After entering data in Google Docs, the students use an interactive Excel spreadsheet to compare their results to the best-fit line obtained by linear regression (pre-built into the spreadsheet for novices). In the spreadsheet, they further explore the model to gain an understanding and examine the influence of scatter (error) in the data and material density.

Keywords
extrapolation, mathematical modeling, measurement, simulation

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1. Introduction

Introducing students to mathematical modeling and simulations is an important science process skill that can enhance discovery learning [1] and fosters developing a conceptual understanding [2]. This activity presents a simple introductory measurement experiment that introduces students to aspects of mathematical modeling using interactive animated spreadsheets. The activity collects data using a balance by massing a bolt with from one to five nuts on it, and then students plot a graph on graph paper and using a ruler and perform an eye-ball fit of the fairly linear data. From the hand-drawn line, they determine the equation of the line. This is a nice use and review of previously acquired algebraic skills. From the graph, they estimate the mass of the bolt by extrapolation (the y-intercept). After they have an estimated (indirect) mass, the students mass the bolt directly on the balance. This allows them to calculate the percent error in their indirect determination.

In essence students must make careful measurements on the balance and construct a graph where the data is plotted correctly. From the graph, they must fit the data with a line and determine its equation. Manual graphing skills force students to scale axes correctly and read the scales to plot points on a coordinate plane. Students share their data via Google Docs and participate in a subsequent class discussion.

After data collection and sharing, the model students manually developed is checked by comparison using a pre-built "just add data" spreadsheet that delivers a best-fit line by linear regression and provides estimates of the errors in the intercept, and slope as well as the coefficient of determination, r^2. Knowledge of linear regression is not needed to make this comparison and the use of r^2 is explained on the spreadsheet. Students further investigate random error and the effect of material density on the model. Using interactive Excel spreadsheets to enhance discovery learning and foster conceptual understanding has been discussed elsewhere [3, 4]. This activity is used in our first semester general chemistry course in about week three to introduce students to modeling.

2. Experimental Method

Students work in groups of four and obtain a bolt and 5 nuts from the instructor. They determine the mass of the bolt with one nut on it using a laboratory balance, and record the value to the nearest 0.01 gram. After

recording the mass, they add a nut to the bolt and record the mass of the bolt with two nuts on it. Students repeat the process until they have recorded the mass of the bolt with 5 nuts on it.

We use electronic balances accurate to two decimal places, and have included a typical set of student data in Table 1. Those wishing to do a slightly more sophisticated experiment could have the students calibrate the balances with a set of standard weights, but in any case calibration should be done before the experiment to check balances for any offsets or linearity errors.

Table 1. Total Mass of Bolt and Nuts

Number of Nuts on the Bolt	Mass (g)
1	38.49
2	45.27
3	52.04
4	58.83
5	65.69

Future work will involve use of beam balances (mechanical) to introduce more scatter in the data. These results will be compared to the electronic balance data as discussed in section four.

3. Graphing and Determining the Equation of a Line

Students plot the data collected on graph paper, and are required to use a grid with at least 10 blocks per inch. Although graph paper is not as available as it once was, it can be printed from a number of websites. Once the data is plotted they draw the best-fit line through the data points and extend the line to the y-axis to determine the y-intercept; which gives the estimated mass of the bolt from extrapolation. Using two points on the line, they determine the slope of the line, which yields the average mass per nut. Using the slope and y-intercept they determine the equation of the line, which they write in terms of the variables studied (not x and y). After the graph and estimate, we have them mass the bolt directly on the balance and calculate the percent error (%error = (indirect value - direct value)*100/direct value).

4. Collaborative Sharing of Data in Google Docs

At this point after acquiring and analyzing the data in a manual fashion, students enter their data in a Google form (link provided to students), which is part of the Google Docs spreadsheet application. They enter their five masses of the bolt with nuts, the slope and y-intercept determined, and the direct mass of the bolt. The Google Docs spreadsheet is projected onto a screen in the laboratory to allow viewing of the group data as it is entered (Figure 1). Setting up the form and collecting the data is easy and straight forward in Google Docs and the data can be exported to Excel [5]. The order of the information on the Google form is stored in the columns of the Google Doc spreadsheet.

Once the class data is entered, a discussion of the results is conducted as if each group was a collaborator at a different institution. Future plans are to include the built in chat feature available in Google Docs. From the extrapolated bolt mass column in Figure 1, a good discussion of significant figures could arise. Part of the collaborative discussion includes the manufacturing uniformity of the nuts and bolts as observed in the data. The students can only view the spreadsheet, while instructors have editing privileges.

It is interesting that students tend to under estimate the extrapolated mass as seen by the negative percent error. The use of Google Docs was an experience that was new to our students and was well received.

Timestamp	Instructor Code	Group Number	Mass With One Nut	Mass With Two Nuts	Mass With Three Nuts	Mass With Four Nuts	Mass With Five Nuts	Extrapolated Bolt Mass	Mass Of Bolt Using Balance	Model Slope	%error
2/9/2011 20:46:12	TED-1	1	38.3	44.99	51.72	58.37	65.07	31.62	31.68	6.69	-0.19
2/9/2011 20:49:18	TED-1	3	38.49	45.27	52.04	58.83	65.69	31.89	31.74	6.83	0.47
2/9/2011 20:54:44	ted-1	6	38.46	45.29	52.11	58.92	65.74	31.1	31.68	6.82	-1.83
2/9/2011 21:03:19	ted-1	4	38.55	45.34	52.16	58.95	65.81	31	31.68	6.82	-2.15
2/9/2011 21:10:31	TED-1	2	38.64	45.47	52.36	59.13	65.95	31	31.75	6.88	-2.36
2/9/2011 21:14:18	Ted-1	5	38.45	45.26	52.09	58.91	65.75	30	31.63	6.83	-5.15
6/9/2011 9:50:25	ted-2	2	38.48	45.29	52.09	58.9	65.72	31	31.66	6.81	-2.08
6/9/2011 9:52:49	TED-2	3	38.45	45.22	52.07	58.88	65.67	30.67	31.7	7	-3.25
6/9/2011 9:55:56	TED-2	4	38.63	45.4	52.2	58.9	65.79	31.82	31.8	6.77	0.06
6/9/2011 9:57:04	TED -2	5	38.65	45.48	52.37	59.19	65.97	31.82	31.75	7	0.22
6/9/2011 10:02:25	TED-2	5	38.44	45.26	52.08	58.93	65.75	31.5	31.61	6.75	-0.35
6/9/2011 10:11:52	TED-2	1	38.38	45.01	51.66	58.38	65.07	31.71	31.69	6.66	0.06

Figure 1. Student Data in Google Docs Spreadsheet

5. Modeling with an Interactive Excel Spreadsheet

Next the activity determines how well students did with developing their model from their hand-drawn graph. They open an interactive Excel spreadsheet and enter their raw data along with the values of the slope

and y-intercept determined by the hand-drawn graph. Figure 2 is a screen capture of the spreadsheet, and the data in the yellow cells is student data. The "add linear model to graph" check box is then selected to generate a linear regression fit; and an r-squared value is given, along with an explanation to judge the goodness-of-fit. Students quickly get a sense of how well they did with the manual graphing and their collection of data. Using this interactive spreadsheet, students explore and discover behavior of their model. This is a good case where the physical model and mathematical model are relatable by students.

Access via hyperlinks to the Google Form (readers can view) and accompanying spreadsheet with all the data can be found on the Excel spreadsheet mentioned above on the "Data for Google Docs" tab, which is how students access it as well.

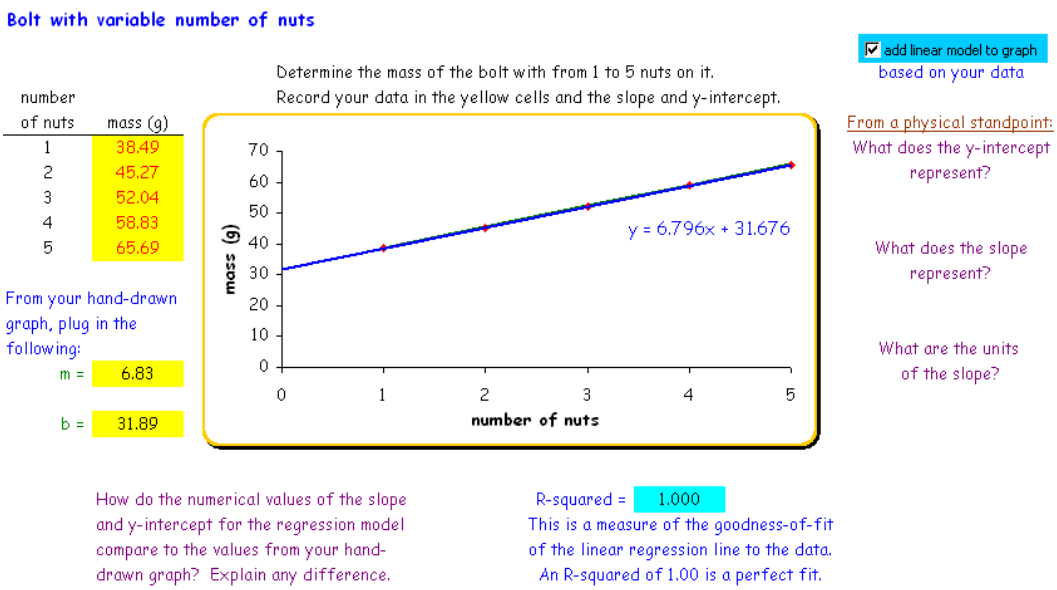

Figure 2. Add Your Data Tab

Students do not need to know how linear regression works at this point, only that it products the best-fit line through the data. Discussion of linear regression can be done at a later time. Knowing how to judge the goodness-of-fit of the linear regression is a topic for further discussion. If students are careful, they get data that illustrates the uniformity of manufacturing of the nuts and bolts (a very high r-squared value). We also want them to understand the meaning of the model – slope is the average mass of the nuts and the y-intercept is the mass of the bolt. The "generate a model" tab illustrates this further (Figure 3) by using the

"number of nuts" spinner to see that the mass increases in increments of the mass of the nut. After this portion of the laboratory activity is concluded, effects of experimental error are explored.

6. Experimental Error

Students see that the bolt and nut masses generate the equation. Next random error was introduced in the data to explore the effects of experimental variation from the balance or mass variation in the nuts and bolt. On the "generate a model" tab (Figure 3) below the graph, a scroll bar is available that adds random noise to the mass data using the RANDBETWEEN function available in Excel. Students explore what happens as the noise is increased, and discover that the data points become more scattered (r-square decreases) and the slope and y-intercept vary randomly. Students may need to be reminded to use mathematical language in their write-up, and that the noise influences the data points which then influences the regression line and fit. For further discussion on errors, see [6].

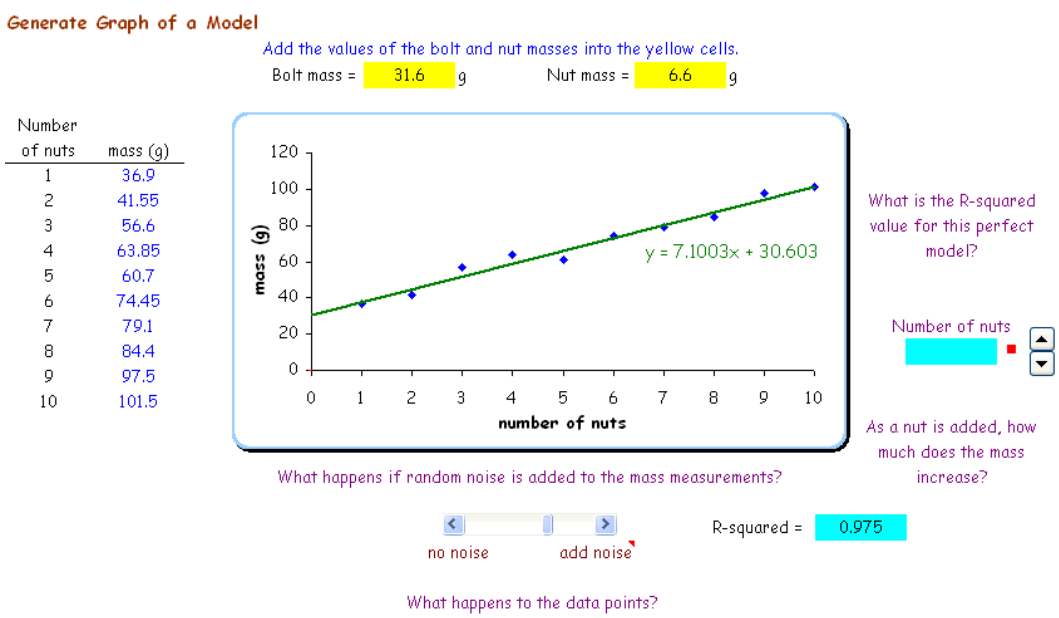

Figure 3. Generate a Model Tab

7. More Multivariable Aspects

Our system is fairly simple but as students progress they will come across more and more complicated systems to study. Again we want to be sure that they understanding the model they developed as well. So another variable is added into the model, a change in material density and its effects on the model can be explored. On the "material density" tab,

students can select seven other materials in place of stainless steel using a dropdown menu to see how the model varies (Figure 4 shows aluminum compared to stainless steel), and they can also mix materials as well. They can add a stainless steel comparison line to help interpret the changes in the model when density changes. This tab adds a more multivariable approach, which is typical of most real-world models, and usually lacking in most college science laboratory single variable pair experiments (measure only one dependent-independent variable pair - while holding all others constant).

Additional variation could be introduced by filing some of the nuts at random to produce more noise in the data. It would also be advisable to carefully select bolts that are close to the same mass for comparison between groups.

Students could also be asked if the mass of a stainless steel bolt determined by extrapolation was done by two students, one using stainless steel nuts and other using aluminum nuts, how would the final bolt mass compare. The list of possible higher-order thinking questions is unlimited. We are looking for students to develop a conceptual understanding of this mathematical model.

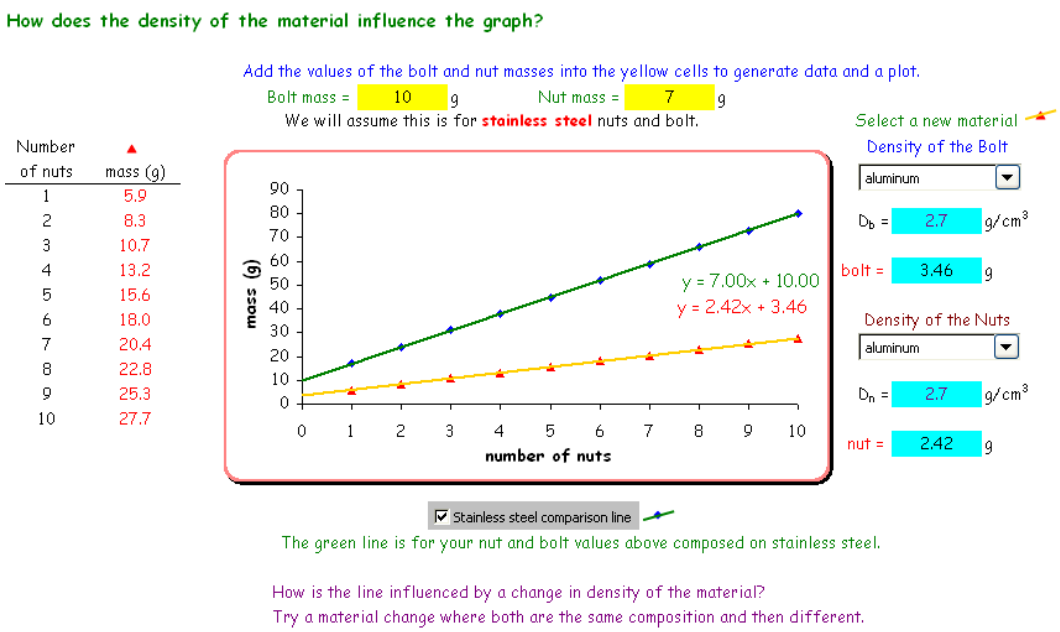

Figure 4. Material Density Tab – Compares Aluminum to Stainless Steel

8. Student Feedback

Students like the interactive nature of the spreadsheets and the visual aspects for enhancing concept understanding [3, 4]. Table 2 gives the results of anonymous student feedback from two different classes of general chemistry I. Results were very positive and show improvement on the second (summer) trial. This could be due to revisions after the initial trial or more diligent summer students.

Table 2. Feedback from Post-activity Survey of Students

Concern	Spring 2011 (n = 24 students)	Summer 2011 (n = 22 students)
Instructions Clear and Unambiguous	4.33	4.77
Activity Was Informative About Extrapolation	4.38	4.68
Size of Work Groups Appropriate	4.54	4.81
Activity Rated as a Learning Experience	4.13	4.63
Activity Overall Rating	4.20	4.59

Written comments from students were positive and demonstrated that this activity can immerse students into mathematical modeling. Students commented this activity illustrated the technique of extrapolation using a simple, readily understandable approach; they were able to focus on the technique of data extrapolation with an easily understandable experiment.

Table 3. Selected Student Comments

Positive comments:
- Google Docs: We could look at the data from other groups in real time and see the variations in the data. - Within Group Interaction: The discussions among group members were very helpful. - Technology/Innovation: It brought another line of innovation to a first semester chemistry course.

- Understandability: (1) Made extrapolation clear, will not forget the concept. (2) Explaining extrapolation and how to use it in simple terms. (3) Allowed "us" to see that extrapolation really works. - Critical Thinking: It helped enhance critical thinking skills.
Constructive Criticism:
- Activity Length: Lab too long for time allocated, shorten the lab activity. - Equipment: More equipment "to go around." - Repetitive: Some questions seem repetitive in concept. - Concept Review: Conduct a better review of the regression line and linear equations. - Enhance Variables: More activities using different variables in exploring extrapolation. - Question Format: Number the questions (Note: This has been done.)

Table 3 summarizes some select positive and constructive comments. Overall, students were very engaged and enjoyed the collaborative analysis and discussion using Google Docs, and eager to compare and share results. Many students commented how this approach could be used in other collaborative efforts such as editing documents in real time during a conference call with fellow collaborators in their respective career fields.

9. Some Final Thoughts

This very simple experiment gets students into mathematical modeling in our first semester of general chemistry course very early in the semester. It introduces students to a collaborative effort under controlled conditions, mimicking the common practice of many scientists. It also reviews many algebraic skills that are used throughout the course and brings an engaging pedagogy to the laboratory using a multivariable approach. Students are introduced to data handling errors and how they can influence models, something that is very important and not well covered in beginning courses (but can be as seen here).

The interactive spreadsheet is all done using computational formulas and logical (IF) statements in Excel. As a follow-up experiment, we are developing an activity and spreadsheet based on the position of an errant

nut of different density being included with the five added nuts for second semester general chemistry.

10. References

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