

11-22-2011

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Recommended Citation

Sugden, Stephen (2011) The Slumbarumba Jumbuck Problem, *Spreadsheets in Education (eJSiE)*: Vol. 5: Iss. 1, Article 2.
Available at: <http://epublications.bond.edu.au/ejsie/vol5/iss1/2>

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The slumbarumba jumbuck problem

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Abstract

The potential for simple linear relationships arising from a computer game to build student modelling and "word problem" skills is explored. The fundamental capability of the spreadsheet to tabulate and graph possible solutions is used to lay bare the problem structure for the students.

Keywords: inequalities, simultaneous equations, degrees of freedom, spreadsheet modelling.

1 Introduction

A computer game, apparently based on ideas and language taken from the classic Australian bush song *Waltzing Matilda*, gives rise to some interesting exercises in elementary linear algebraic relationships. These exercises require modelling of linear equality and inequality relations and seem to be suitable for students at middle-school level to early tertiary level in Australia. The origin of the problem is uncertain, and an Internet search in September 2011 yields only two incomplete references to it [6], [7]. The following statement comes from a school assignment set for Grade 10 students at a Gold Coast high-school in 2005.

1.1 Problem statement

In the computer game *Slumbarumba Jumbuck*, the player helps Matilda to stow as many jumbucks as possible in her tuckerbag before the troopers arrive. The computer keeps a record of both the score for each game and the average of that score and the two previous scores, called the 3-score average.

Q1 Steve plays a sequence of seven games. His five 3-score averages are 22, 14, 17, 16 and 18, in that order. That is, the average score for Steve's first, second and third

games is 22; the average score for his second, third and fourth games is 14; etc. If Steve scored 17 in his second game, what did he score in his fifth game?

Q2 Steve plays a new sequence of seven games. This time, his five 3-score averages are 12, 17, 19, 18 and 14, in that order. No individual score is below 10.

- (a) What is Steve's greatest possible score? In which game(s) could it have occurred?
- (b) If the average of Steve's seven scores is a whole number, what could his scores have been in each of the seven games?

2 Modelling skills

My principal interest in this problem set is to investigate its potential to assist with the strengthening of mathematical modelling skills of high-school students, and perhaps also for beginning tertiary students. Indeed, for this purpose, I believe it could be profitably used at tertiary level; for example, in my *Elementary Mathematics* or *Spreadsheet Modelling* classes at Bond University, Australia. What modelling skills may be developed here? The present problem, though rather simple, nevertheless affords a number of opportunities for the teacher to point out useful modelling strategies, procedures and heuristics. These are now described, and the benefits of using spreadsheets to model the problem are outlined.

2.1 Getting started

The first opportunity for students is simply practice at reading a *word problem*, identifying the unknowns and then making some attempt to write an algebraic formulation of the problem. In the author's experience, few tertiary students have the necessary skills to make even this first step. Certainly, this is true at Bond University. But we know that there are really no shortcuts to this: in order to develop problem-solving skills, the "math doctor" recommends a steady diet of reasonably-tractable *word problems*. As one deals with more and more of these, skills gradually develop. Some general strategies and heuristics may be given and two well-known books by Dromey [4] and Polya [5] consulted. A good summary of Polya's work may be found online at [10].

After reading the problem statement a few times, our next step is to choose the unknowns (sometimes called *decision variables*, especially in linear programming and related fields). For our jumbuck problem, let x_k represent the score on game k for $1 \leq k \leq 7$. Even a cursory understanding of the problem now allows us to write the system of equations (1). As we progress through the steps of solving the various parts of the jumbuck problem, it is evident that the ability of spreadsheets to tabulate possibilities is very handy indeed. We also find that some modest amount of simple algebraic manipulation (just linear in two parameters) is required. If appropriate to the class, this may be used by the teacher

as a gentle introduction to algebra and its usefulness, with strong spreadsheet support for representation and visualization of each of the three subproblems (questions 1, 2a, 2b).

3 Technology

What technology is appropriate for the present problem? I have somewhat preempted my remarks here and mentioned spreadsheets already. But after all, this is an article in the journal *Spreadsheets in Education!* As readers of my other published works may have discerned, I eschew graphics calculators (GCs). I acknowledge that these devices have grown very powerful in recent years and many boast *computer algebra systems* (CAS), but I still believe GCs represent a relatively poor choice for modelling. My choice for most modelling and simulation is Excel, but I do use Wolfram's *Mathematica* and write 3GL code when necessary. Interestingly, I almost never find either of these to be necessary. Much of my research modelling is also done in Excel, and that is by choice, not necessity. I have no great love for the language, but I sometimes supplement my Excel models with some of Microsoft's *Visual BASIC for Applications* (VBA) code as it is usually adequate and very convenient, albeit slow. Even so, I almost never need anything beyond Excel when creating models for my classes. Why?

A large part of the reason is that the mathematics I teach at Bond is relatively unsophisticated, first-year material. Most of the students have problems with even middle-school algebra, so the extent to which I can press basic algebraic techniques into service for analyzing and solving problems is severely limited. I also believe that the approach where I use a software tool (Excel) with which the students already have some familiarity, and which they will almost certainly be required to use in the workforce, has much to recommend it. Similar but more detailed comments may be found in [3], [8]. Excel is already installed on essentially every university and student computer. The spreadsheet medium gives immediate feedback on modelling steps and it lends support for quick tabulation and graphing. In short, it provides just what we need to model and solve each part of the present "jumbuck" problem. An additional positive is that students gain valuable experience with spreadsheet modelling; a very useful and highly sought-after skill when job-hunting. Spreadsheets are used extremely widely in scientific, engineering and business worlds, and to a far greater extent than *graphics calculators*. In fact, there is recent evidence to suggest that *spreadsheets, not graphics calculators*, form a good basis for a common language when mathematicians, scientists, engineers and business people need to communicate in quantitative terms. In this connection, the views and very recent first-hand experience of Tonkes are highly relevant.

“In the finance and energy industries, Microsoft Excel is an invaluable form of communication between professionals to convey results, to communicate illustrative models, and transport and display summary (and detailed) data. Key to the utility of Excel is that spreadsheets are ubiquitous across different disciplines: accountants build financial models and budgets, traders keep track of

forecasts and portfolio positions, and engineers and operational personnel use spreadsheets for modelling physical processes."

"It has been found that the commonality amongst a diverse range of workshop participants is their understanding of Excel, and it forms an ideal mechanism to communicate technical and mathematical concepts."

"Inevitably, mathematical, technical, and algorithmic concepts arise in the delivery of the workshop syllabus. The mathematical abilities of the audience cover a wide spectrum of sophistication. However, our experience is that workshop attendees are universally familiar with spreadsheets. With spreadsheet software as a modelling platform, we are able to construct simple models to convey complex concepts. The use of spreadsheets throughout industry is possibly far more pervasive than researchers in the educational arena suspect." [9]

3.1 Spreadsheet models help with illustration of inequalities

In everyday life, all quantities are limited and finite. Inequalities inevitably occur in daily existence. When doing mathematical modelling, we can only pretend that they do not exist for just so long, and sooner or later we must learn to deal with them. Experienced mathematics teachers at all levels know that students often have great difficulties in dealing with inequalities. Inequalities often require us to estimate, and even professional mathematicians and scientists may have difficulties—*inequalities are hard* [2]. At least for elementary problems such as the present one, I argue that the spreadsheet modelling environment can help significantly to illustrate the use of linear inequalities to solve problems. The present jumbuck problem serves as a gentle introduction to linear inequalities in two unknowns, and also to the more advanced topics of linear programming and integer programming. For this work, the spreadsheet is also useful for tabulation and solution via the *Solver* add-in.

4 Spreadsheet solution sketches

4.1 A solution sketch for question 1

$$\begin{array}{rcl}
 x_1 + x_2 + x_3 & & = 3 \times 22 \\
 & x_2 + x_3 + x_4 & = 3 \times 14 \\
 & & x_3 + x_4 + x_5 & = 3 \times 17 \\
 & & & x_4 + x_5 + x_6 & = 3 \times 16 \\
 & & & & x_5 + x_6 + x_7 & = 3 \times 18
 \end{array} \tag{1}$$

At first sight, we have five equations in seven unknowns. The teacher familiar with the basics of linear algebra therefore knows that this leaves two degrees of freedom, assuming no degeneracy. However, in this author's experience, the vast majority of students are *not* aware of this. At this point it helps to re-cast the model in terms of two parameters, a

Table 1: Model for Q1, for arbitrary score b in the second game

x_1	x_2	x_3	x_4	x_5	x_6	x_7
a	b	$66 - a - b$	$a - 24$	$b + 9$	$63 - a - b$	$a - 18$

and b , being the values of x_1 and x_2 respectively. As noted, this problem has merit in helping students develop modelling skills. Those with relatively mature problem-solving and modelling skills will realize early that it makes sense to multiply each of the five averages by three. Then the sums of scores (three at a time) may be written simply as in the system of equations labelled (1). The very simple form of the equations (tridiagonal) is then easily noticed. However, it is likely that only the teacher may know this terminology, and also the concept of "degrees of freedom". Perhaps even the teacher will not know it. For a class in elementary linear equations at middle-school level, to which this problem is targeted, the concept of an underdetermined linear system with degrees of freedom is likely to be unfamiliar. In such cases, the teacher may help by suggesting that two parameters be set up, as shown in Table 1. The expressions for x_3 through x_7 may be obtained by some limited algebraic manipulation as follows¹.

1. We start by restating that $x_1 = a$ and $x_2 = b$. Our aim is to express each unknown x_3 through x_7 in terms of the parameters a and b .
2. The first of equations (1) may now be used to obtain x_3 . We have $x_1 + x_2 + x_3 = 66$, so $x_3 = 66 - x_1 - x_2 = 66 - a - b$.
3. The second of equations (1) may now be used to obtain x_4 . We have $x_2 + x_3 + x_4 = 42$ so $x_4 = 42 - x_2 - x_3 = 42 - b - (66 - a - b) = a - 24$.
4. The third of equations (1) may now be used to obtain x_5 . We have $x_3 + x_4 + x_5 = 51$ so $x_5 = 51 - x_3 - x_4 = 51 - (66 - a - b) - (a - 24) = b + 9$.
5. The fourth of equations (1) may now be used to obtain x_6 . We have $x_4 + x_5 + x_6 = 48$ so $x_6 = 48 - x_4 - x_5 = 48 - (a - 24) - (b + 9) = 63 - a - b$.
6. Finally, the fifth of equations (1) may now be used to obtain x_7 . We have $x_5 + x_6 + x_7 = 54$ so $x_7 = 54 - x_6 - x_5 = 54 - (63 - a - b) - (b + 9) = a - 18$.

If we now make use of the information that $x_2 = b = 17$, Table 2 results. From this it is clear that $x_5 = 26$, which is our answer for question (a). Notice that this is independent of the first score $x_1 = a$.

¹For an alternative spreadsheet model which solves question one and which requires no algebra at all, see Section 4.4.

Table 2: Model for Q1, for score 17 in the second game

x_1	x_2	x_3	x_4	x_5	x_6	x_7
a	17	$49 - a$	$a - 24$	26	$46 - a$	$a - 18$

Table 3: Model for Q2(a), for arbitrary score b in the second game

x_1	x_2	x_3	x_4	x_5	x_6	x_7
a	b	$36 - a - b$	$a + 15$	$b + 6$	$33 - a - b$	$a + 3$

4.2 A solution sketch for question 2a

Again, we let the seven scores be x_1 to x_7 respectively.

$$\begin{aligned}
 x_1 + x_2 + x_3 &= 3 \times 12 \\
 x_2 + x_3 + x_4 &= 3 \times 17 \\
 x_3 + x_4 + x_5 &= 3 \times 19 \\
 x_4 + x_5 + x_6 &= 3 \times 18 \\
 x_5 + x_6 + x_7 &= 3 \times 14
 \end{aligned} \tag{2}$$

Adopting an approach similar to the previous question, again write $x_1 = a$ and $x_2 = b$ and Table 3 results.

It is often difficult for students to make use of inequality information. In this question, we are told that all scores are at least 10. How can we make use of that extra piece of information here? We are told $x_k \geq 10$, for $1 \leq k \leq 7$. From the expression $x_3 = 36 - a - b$ (column three of Table 3), we deduce that $a + b \leq 26$. This is so, because x_3 cannot go below 10. However, we can do better than this, since we have a similar situation for x_6 . By the same kind of reasoning, we see that $a + b \leq 23$. The relationship $a + b \leq 26$ is made redundant by new information $a + b \leq 23$, that is, the latter inequality subsumes the former. This phenomenon occurs routinely in linear programming modelling and here we have a simple but useful illustrative example of a "redundant constraint". Since $a \geq 10$ and $b \geq 10$, we also have $a \leq 13$ and $b \leq 13$. At this point it is probably best to ask the students to graph the three inequality relationships $a \geq 10$, $b \geq 10$ and $a + b \leq 23$. If we do this in the Cartesian plane with a and b playing the roles of x and y we obtain something like Figure 1.

This is, of course, conveniently done in Excel. Also in Excel we may tabulate $a + b$ for $10 \leq a \leq 15$ and $10 \leq b \leq 15$. Using conditional formatting to automatically highlight those cells not exceeding 13 in a tabulation of the function $a + b$, it soon becomes clear that neither a nor b can exceed 13 (Figure 2). We have exactly ten possibilities, corresponding to the ten lattice points of Figure 1, and in relationship 3 the ordered pairs are deliberately

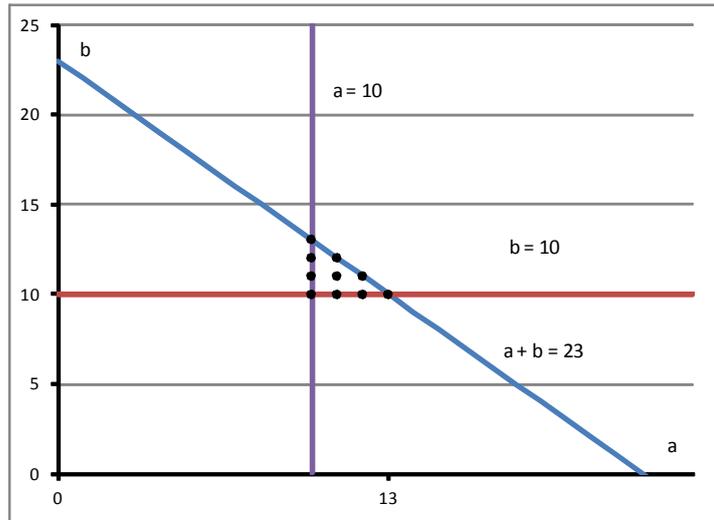


Figure 1: Graphing linear inequalities

presented in the same triangular arrangement as the lattice points of Figure 1.

$$(a, b) \in \left\{ \begin{array}{l} (10, 13), \\ (10, 12), (11, 12), \\ (10, 11), (11, 11), (12, 11), \\ (10, 10), (11, 10), (12, 10), (13, 10) \end{array} \right\} \quad (3)$$

To finally obtain the answer to Question 2a, we now construct a table similar to Table 3, but replacing each expression in that table by its maximum over the set shown in (3). This process yields Table 4 and tells us that the greatest possible score is 28, occurring in the fourth game.

a \ b	10	11	12	13	14	15
15	25	26	27	28	29	30
14	24	25	26	27	28	29
13	23	24	25	26	27	28
12	22	23	24	25	26	27
11	21	22	23	24	25	26
10	20	21	22	23	24	25

Figure 2: Conditional formatting in Excel used to highlight feasible pairs (a, b)

Table 4: Model for Q2(a), for arbitrary score b in the second game

x_1	x_2	x_3	x_4	x_5	x_6	x_7
13	13	16	28	19	13	16

Table 5: Relationships for question 2b

x_1	x_2	x_3	x_4	x_5	x_6	x_7
12	b	$24 - b$	27	$b + 6$	$21 - b$	15

4.3 A solution sketch for question 2b

For this question, we are told that the average of the seven scores is an integer. Thus:

$$\sum_{k=1}^7 x_k \equiv 0 \pmod{7} \tag{4}$$

Using the expressions from Table 3, we have:

$$\begin{aligned} 36 + a + 15 + b + 6 + 33 - a - b + a + 3 &\equiv 0 \pmod{7} \\ 93 + a &\equiv 0 \pmod{7} \\ a &\equiv 5 \pmod{7} \end{aligned} \tag{5}$$

Since $10 \leq a \leq 13$, the only possibility, consistent with $a \equiv 5 \pmod{7}$, is $a = 12$. We arrive at Table 5. From the expression for x_6 there and the fact that $10 \leq b \leq 13$, it is seen that b may only be 10 or 11, else x_6 would fall below 10. Thus, there are only two possible sequences of scores, corresponding to the values $b = 10$ and $b = 11$ respectively. For $b = 10$ the scores are 12, 10, 14, 27, 16, 11, 15 and for $b = 11$, they are 12, 11, 13, 27, 17, 10, 15. In both cases, the greatest score is 27.

4.4 An alternative spreadsheet model for question 1

We now describe an alternative Excel model to solve question one of the jumbuck problems. This model requires no algebra whatsoever, and thus may be suitable for pre-algebra students. Here are the steps to construct it, using Excel 2007.

1. In A1 and B1 respectively place the headings "Game #" and "Score" on a fresh worksheet.
2. In A2 through A8 place the numbers 1 through 7. These are the game numbers for reference purposes only and do not appear in any formulas.

3. We now place a slider (horizontal scrollbar) on the worksheet. In order to do this, one must have the Developer tab visible on the Excel ribbon. If absent, this may be switched on from Excel Options, Popular. Go Insert, Form Controls, Scroll Bar and drag to create the scrollbar covering cell C2. Right-click on the scrollbar, then Format Control and make the cell link $\$B\2 (second game score). While this format Control dialog is still open set the minimum value to 0 and the maximum to 50. Close the dialog. You can now slide the scrollbar and watch the value in B2 change.

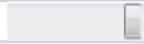
	A	B	C	D
1	Game #	Score		
2	1	49		
3	2	17	3-game average	
4	3	0	22	
5	4	25	14	
6	5	26	17	
7	6	-3	16	
8	7	31	18	
9				

Figure 3: Alternative Excel method for question 1

4. We are told that the score for game 2 was 17, so place this value in cell B3.
5. In cell C3 place the heading "3-game average". Then in C4:C8 place the given averages 22, 14, 17, 16, 18.
6. Now we need to model the requirement of "running averages". The prototype formula, to be placed in cell B4, is $=3*C4-B3-B2$. This expresses the fact that triple the running average in C4 less the two previous score must be the current score. Fill this down to B7 and the model is almost complete.
7. Students may now use the slider (scrollbar) to observe the effect of changing the initial score on the contents of cell B6, which is precisely no effect at all. It becomes very clear from using the slider that, while other values in column B depend on the initial score, the fifth game score, in B6, does not.
8. A nice touch for the model is to employ *conditional formatting* to alert the user to improper values. Select the cells B2:B8 and go Conditional Formatting, Highlight Cells Rules, then Less Than 0, to highlight these cells if they become negative. An example of this appears in Figure 3.

5 Conclusion

An approach to solving three problems related to seven linearly-related unknowns is described. Several problem-solving principles are illustrated, and the desirability of continual practice at word problems for students and middle-school and senior school levels is discussed, with a view to building and strengthening their skill-sets. The important concept of problem parametrization resulting from degrees of freedom in a linear problem is also mentioned. Tabulation of a modest set of possibilities plus simple algebraic and graphical analysis of linear inequalities in two variables, with support from spreadsheet models is also recommended for the "jumbuck" problem considered here. Finally, the well-known potential of the electronic spreadsheet to support mathematical modelling and transition to algebra [1], [11] is again illustrated.

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