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The Use of Microsoft Excel to Illustrate Wave Motion and Fraunhofer Diffraction in First Year Physics Courses

Garry Robinson
The University of New South Wales, Australian Defence Force Academy, g.robinson@adfa.edu.au

Zlatko Jovanoski
The University of New South Wales, Australian Defence Force Academy, z.jovanoski@adfa.edu.au

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Abstract
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Keywords
waves; interference; diffraction; interactive simulations; Excel

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The Use of Microsoft Excel to Illustrate Wave Motion and Fraunhofer Diffraction in First Year Physics Courses

Garry Robinson and Zlatko Jovanoski
School of Physical Environmental and Mathematical Sciences,
The University of New South Wales, Australian Defence Force Academy,
Canberra ACT 2600, Australia

July 14, 2011

Abstract

In this paper we present an Excel package that can be used to demonstrate physical phenomena in which variables may be automatically adjusted in real-time. This is accomplished by interrogating the system clock through the use of an appropriate macro, and using the clock reading to update the relevant variable. The package has been used for a number of years in first year physics courses to illustrate two phenomena: i) waves, including travelling waves, standing waves, the addition of waves and the interference of waves in general, and also Lissajous figures, and ii) Fraunhofer diffraction and the effects of varying such quantities as the wavelength of the incoming light, the number of slits, the slit width and the slit separation. A number of illustrative examples, generated by the package and taken from a first year physics course, are presented graphically. The package, which is available for downloading from the web, may be used interactively by the student and is easily modified by them. The use of Excel has the advantage that it is accessible to a much wider audience than if it were written in, say, Matlab. We envisage that it may be useful for first year university courses in wave motion and optics, and may also be useful in physics courses in the last year of secondary school. The package has been tested under Excel 2003, 2007 and 2010, and runs satisfactorily in all three versions.

Key words: waves; interference; diffraction; interactive simulations; Excel

1 Introduction

Today's students seem to learn in a different manner from the learning methods of the students of a few years ago. In particular they appear to benefit greatly from, and respond to, "hands on" interactive methods. Two subjects which are particularly suitable for such techniques are the subjects of wave motion and of diffraction. One of the authors (GR) has lectured in these subjects in first year physics courses for many years. While many very good interactive packages for these purposes are available either on the web or accompanying textbooks in the subject, they are usually presented in such a way that the student cannot easily modify them, or they are presented in a language such as MATLAB, which may not be available to everyone. Some users do not have available such specialised packages, or worse, may lose access to them if they move from institution to institution. Even if a stand-alone executable can be made available, this still lacks the flexibility of having access to the source. Here we present an interactive package, written in the virtually universally available Microsoft EXCEL, which has been used for demonstrating waves in general and also the phenomena of Fraunhofer diffraction. This package has been used for about seven years in first year university physics courses, and each year when the displays are first presented in class, a quantum increase in the students' interest has resulted.

In this paper we present details of the EXCEL package together with some illustrative simulations. The structure of the paper is as follows. In §2 we present a brief summary of the theory of waves (§2.1) and of Fraunhofer diffraction (§2.2). In §3 we give details of the EXCEL package and its use, specifically the use of real-time simulations and the associated macro (§3.1), and the EXCEL control panels (§3.2). In §4 we present some illustrative simulations for waves (§4.1) and Fraunhofer diffraction (§4.2). In §5 we include some of our experiences in the classroom and finally in §6 we provide a brief summary of this work and some conclusions.

2 Theory

The basic theory of wave motion and of Fraunhofer diffraction has been very well documented over the years and may be found in any current introductory physics text (e.g., Halliday, Resnick and Walker [1]; Serway and Jewett [2]; Young and Freedman [3]). However, with the aim of making the present paper self-contained, we include here the briefest of summaries of the relevant equations and their implications.

2.1 Waves

Provided we limit ourselves to one-dimensional non-dispersive waves (waves whose shape does not change as the wave progresses), any function, g, of the form

$$y\left(x,t\right)=g\left(x\pm vt+\phi\right), \tag{1}$$

represents a travelling (transverse) wave. This is the equation of a wave travelling in the $\mp x$ direction with velocity v and arbitrary phase ϕ. The key point here is that the

position variable x, and the time variable t, must occur in the combination $x \pm vt$, or be able to to be put in this form, for the equation to describe a travelling wave.

The propagation or phase velocity v, is related to the wavelength λ, frequency f, angular frequency ω and (angular) wavenumber k, by the relation

$$v = \lambda f = \frac{\omega}{k}, \tag{2}$$

where $\omega \equiv 2\pi f$ and $k \equiv (2\pi)/\lambda$.

A very important principle in wave motion, and an important point for the demonstrations presented here, is the principle of linear superposition. For the one-dimensional case, it may be stated as follows: "*At any point in a medium subject to more than one wave disturbance, the resultant disturbance is the algebraic sum of the separate disturbances.*" Thus, for example, if the medium is subject to two sine waves, travelling in opposite directions, of amplitudes A_1 and A_2 respectively, with different angular wavenumbers, angular frequencies and phases, the resultant wave amplitude is given by

$$y(x,t) = A_1 \sin(k_1 x - \omega_1 t + \phi_1) + A_2 \sin(k_2 x + \omega_2 t + \phi_2). \tag{3}$$

For the transverse wave of equation (3) a very important property of the wave is the transverse (particle) velocity and acceleration, given respectively by

$$v_{\mathrm{T}}(x,t) \equiv \frac{\partial y}{\partial t} = -\omega_1 A_1 \cos(k_1 x - \omega_1 t + \phi_1) + \omega_2 A_2 \cos(k_2 x + \omega_2 t + \phi_2), \tag{4}$$

and

$$a_{\mathrm{T}}(x,t) \equiv \frac{\partial^2 y}{\partial t^2} = -\omega_1^2 A_1 \sin(k_1 x - \omega_1 t + \phi_1) - \omega_2^2 A_2 \sin(k_2 x + \omega_2 t + \phi_2). \tag{5}$$

Finally, in one of the simulations (to be discussed in §4.1), we present a representation of a travelling Gaussian wave packet; the equation to this wave packet is

$$y(x,t) = \frac{A \exp\left[-\frac{1}{2}\left\{(x - v_{\mathrm{env}} t - x_{\mathrm{init}})/\sigma\right\}^2\right]}{\sigma (2\pi)^{1/2}} \cos(kx - \omega t). \tag{6}$$

In this equation A controls the height of the Gaussian envelope, v_{env} is the envelope velocity, x_{init} specifies the initial envelope position, σ controls the width of the wave packet (as well as affecting its height) and ω/k gives the phase velocity, v_{phase}.

2.2 Fraunhofer diffraction

The situation for Fraunhofer diffraction is depicted in Figure 1, which shows a plane wave incident from the left on a diffracting screen, in this case two narrow slits, with the interference/diffraction pattern being produced on the screen at right.

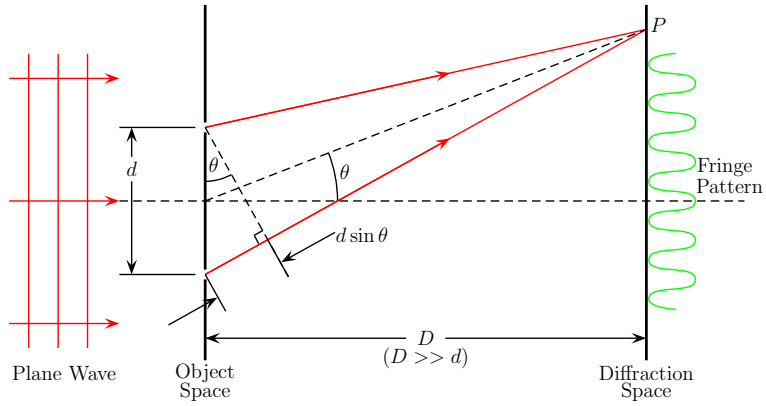

Figure 1: A plane wave from a source at left is incident on a diffraction screen (called "object space") and the diffraction pattern is formed on the screen at right (called "diffraction space" or "Fourier transform space"). In this case the object is two very narrow slits of separation d—this arrangement is the so-called "Young's double-slit experiment". The path difference for waves originating from the two slits and arriving at point P on the screen at right is $d \sin\theta$.

The intensity distribution (the observable quantity) in the Fraunhofer diffraction pattern for the general case of N slits each of width a and (centre-to-centre) slit separation d, for incident wavelength λ, is given by

$$I_\theta = I_0 \left[\frac{\sin N\beta}{\sin\beta}\right]^2 \left[\frac{\sin\alpha}{\alpha}\right]^2 . \tag{7}$$

Here I_0 is the intensity in the $\theta = 0$ direction from one slit, $\beta \equiv (\pi d \sin\theta)/\lambda$ and $\alpha \equiv (\pi a \sin\theta)/\lambda$, where θ is the angle of diffraction (see Figure 1). This equation is rarely presented in introductory physics texts. For detailed information concerning it, as well as diffraction in general, see e.g., Hecht [4] or Bennett [5], and for a definitive treatment of diffraction at an advanced level using Fourier Transforms and Convolutions, see Cowley [6].

Note that for the Fraunhofer diffraction approximation to be valid (the so-called "far-field" situation), both the distance of the source and the screen on which the diffraction pattern is formed from the diffracting aperture must be large compared to the slit separation d and slit width a. In this case we are dealing with plane waves.

For $N = 1$, equation (7) reduces to the single-slit pattern

$$I_\theta = I_0 \left[\frac{\sin\alpha}{\alpha}\right]^2 , \tag{8}$$

and for $N = 2$, it reduces to the double-slit pattern

$$I_\theta = 4 I_0 \cos^2\beta \left[\frac{\sin\alpha}{\alpha}\right]^2 . \tag{9}$$

For two very narrow slits ($a \ll \lambda$, $\sin\alpha \simeq \alpha$) equation (9) yields the familiar Young's double-slit result

$$I_\theta = 4I_0 \cos^2\beta, \tag{10}$$

as portrayed in Figure 1.

The first term in square brackets in equation (7) is usually referred to as the "interference term", and the second term in square brackets is referred to as the "diffraction envelope". Clearly N affects the interference term only and λ affects both terms (through β and α respectively) in an identical fashion, increasing λ broadening the component of the pattern produced by each term. However, to some extent the interference and diffraction terms are independent, the interference term being influenced by the slit separation, d, and is independent of the slit width, a, while the diffraction envelope term is influenced by a and is independent of d. Thus it might be expected that changing d or a independently would alter certain aspects of the overall pattern while leaving other aspects unchanged.

We summarise here the principal characteristics of the diffraction pattern, which may be verified by a close examination of equation (7) (to be discussed in detail in §4.2).

(i) The maxima of the interference term occur at $d\sin\theta = m\lambda$, where $m = 0, \pm1, \pm2, \pm3, \pm4, \cdots$. These are known as principal maxima and m is usually referred to as the "order of interference". The maximum for $m = 0$ (i.e., of order zero) is known as the "central image".

(ii) The minima of the diffraction envelope occur at $a\sin\theta = p\lambda$, where $p = \pm1, \pm2, \pm3, \pm4, \cdots$. (Note that $p \neq 0$; $p = 0$ corresponds to the central diffraction envelope maximum.)

(iii) Between adjacent principal maxima, there occur $N - 2$ weak secondary maxima.

(iv) Suppressed principal maxima or "missing orders" occur if a principal maximum and a minimum of the diffraction envelope occur at the same value of θ, leading to the condition $d/a = m/p$, i.e., d/a is in the ratio of two integers. For example, if $d/a = 3$, for $p = \pm1$, $m = \pm3$; for $p = \pm2$, $m = \pm6$; for $p = \pm3$, $m = \pm9$ etc., and the 3rd, 6th, 9th etc. principal maxima are suppressed.

3 The Excel Package and its use

3.1 Real-time simulation and the associated macro

We have chosen to use the system clock for timing purposes, as this has the advantage, at least in principle, of producing real-time simulations. However, it must be acknowledged that, because of the way PCs as opposed to dedicated machines function, troublesome discontinuous ("jerky") motions may sometimes occur using Excel 2007 on some machines running Windows XP. (This does not appear to occur using Windows 7.) We have not noticed such effects using either Excel 2003 or Excel 2010. Nevertheless we note

that other somewhat simpler methods may in fact be employed to produce a moving animation or to move a point along a curve, and a number of examples of their use have been reported in this journal (Benacka [7], Wischniewsky [8], Oliveira and Nápoles [9], Miller and Sugden [10]).

We are not claiming that all simulations would benefit significantly from being presented in real-time, but rather are suggesting this as an alternative method, which may be useful in certain situations. Aside from the obvious advantage of presenting a display in real-time, such displays have a second advantage in that they run at the same speed on all machines, and can therefore always be compared with the physical behaviour of real objects which they purport to represent, irrespective of the machine characteristics.

While our waves/Fraunhofer diffraction simulations may not be the best examples of the benefits of real-time simulations, an excellent example of a simulation that would benefit is that of the oscillatory movement of a mass-spring system discussed by Oliveira and Nápoles [9]. Although this is a most informative and excellently presented simulation as is, the ability for it to run in real-time would be a useful addition. The actual run time depends on the time discretization interval, the particular machine employed and whether it is run under Excel 2003 or Excel 2007. If this simulation were in fact to run in real-time, one could easily compare it directly with the behaviour of a real mass-spring system with the same mass, spring constant, damping force and external driving force. A second example which would clearly benefit from real-time operation is the projectile motion discussed by Benacka [7], since one could (not so easily) compare the position and time of a real projectile with the simulation.

In summary, we are not suggesting that the benefits in generating real-time simulations on a PC always justify the effort, particularly as there are clearly some limitations to the procedure, but rather that for certain applications there are significant advantages with such displays.

The present simulation originally used the macro "Active-Clock" written by Aaron T. Blood and available on-line [11], and this incremented the time in 1 s intervals. However, this has been extensively modified and extended by a former student and the present authors. It allows time to be stepped in both the forward and reverse directions in increments of 0.1 s, stopped and reset back to zero. It should be emphasised that macros are only used for controlling the time variable, all other processes being performed within the spreadsheet itself. While the use of macros for the entire package no-doubt would result in a much cleaner presentation, their more extensive use has the disadvantage that students not familiar with macros could not easily make modifications.

The macro is written in Excel Visual Basic for Applications (VBA) (see e.g., Birnbaum [12] for an introduction and Kofler [13] for an advanced treatment). In the Appendix we include an extract from the macro code, specifically the section which controls time running in the forward direction. Note that the VBA **"Timer"** function, which returns times read from the system clock to an accuracy of 0.1 s, is the key to its operation. Essentially all that the macro does is to update the time continuously, the zero of time being taken as when the **"Start"** CommandButton (see §3.2 below) is pressed. This time value is written into a single cell (cell "F4") within the spreadsheet.

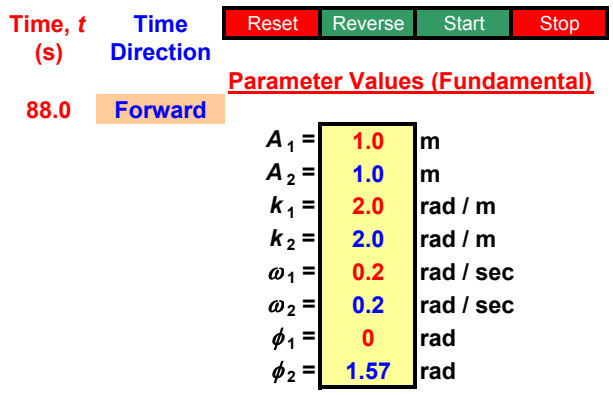

Figure 2: Part of the control panel (slightly modified for illustrative purposes) for one of the wave displays. The icons at the top, labelled Reset, Reverse, Start and Stop respectively, control the motion of time. The parameter values for the two component waves may be altered to suite by changing the values in the yellow boxes. Extra controls are available to, for example, add extra harmonics or switch off either of the component waves or their sum.

3.2 The control panels

Figures 2 and 3 show part of the parameter control section of one of the wave displays and the Fraunhofer diffraction display respectively. Note that only the parameter values in the yellow filled boxes should be changed in the first instance. It may be seen that we have chosen not to use scroll bars. Such controls clearly do have advantages in limiting the range of the variable which they relate to, and hence provide convenience of use. However, if students are forced to change the axis scale (by selecting the relevant axis and either selecting "Auto" scale or entering new limits), they may benefit by learning a little more about the use of EXCEL itself, and from thinking about the physics of the problem.

The time variable is crucial to the operation of the displays and, as noted above, its value is placed within a specific cell in the spreadsheets. For the wave simulations this cell is used directly in a straightforward manner in the relevant equations. For the Fraunhofer diffraction simulations the time variable is used to adjust other parameters (e.g., the slit width a, slit separation d, number of slits N and wavelength λ) by the addition of a term to the parameter selected to be varied with time, with the facility to switch off the time variation for that particular parameter (see Figure 3). For example to adjust a, the appropriate cells contain expressions of the form

```
a(current) = a(initial) * (1 + 0.04*c*t),
```

where the numerical value of 0.04 was chosen to give an appropriate rate of change of a with time, and the constant $c = 0$ or ± 1 is used to switch off the variation or control the direction of the change. In fact any value of c may be used, the absolute value influencing the rate of variation of the parameter. Negative values of c have exactly the same effect

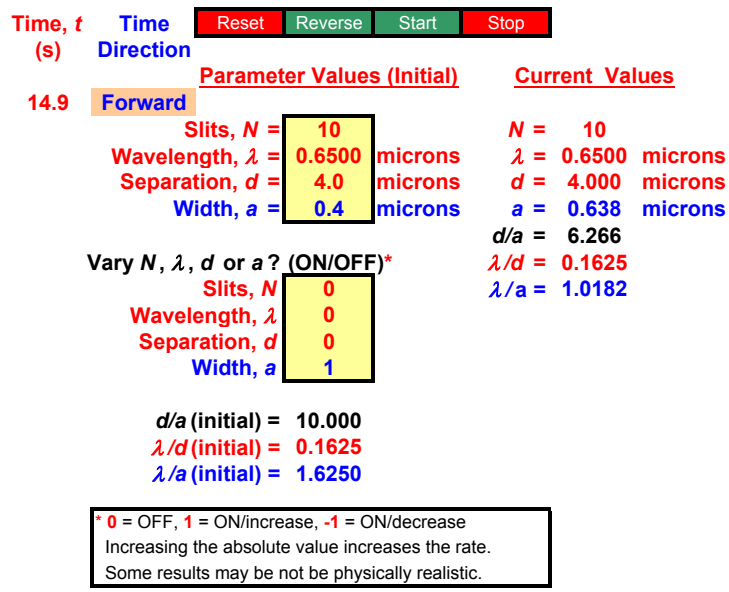

Figure 3: Part of the control panel (slightly modified for illustrative purposes) for the Fraunhofer diffraction pattern display. For the case shown, only the slit width, a, has been allowed to vary with time, as reflected by the current parameter values appearing at upper right, which indicate that a has increased from its initial value of 0.4 μm to 0.638 μm. Note also that d/a, a parameter of considerable significance in Fraunhofer diffraction (see §2.2), has decreased from its initial value of 10.0 to 6.266. The current time value of 14.9 s shown at upper left is of no significance; the time variable is simply used to enable the selected parameter(s) to vary in real-time.

as running time in the negative (**"Reverse"**) direction. For varying the number of slits, because N may take on integral values only, the expression used was of the form

```
N(current) = INT(N(initial) * (1 + 0.2*c*t)),
```

where the numerical value of 0.2 was found to be appropriate, and again $c = 0$ or ± 1.

The package itself consists of twelve separate spreadsheets, the first eleven of which are devoted to waves of various kinds and the twelfth illustrates Fraunhofer diffraction. The waves that may be illustrated include simple sine, square and triangular travelling waves and their sum, Gaussian pulses and Gaussian wave packets, simple ocean waves, beats and Lissajous figures. The Fraunhofer diffraction display enables the slit width a, slit separation d, number of slits N, and wavelength λ, to be varied, and can show three discrete wavelengths on the one display. A detailed "Information Sheet" is included in the spreadsheet. Note that the first of the twelve sheets is in fact a simplified introductory wave display in which the time t and position x may be stepped manually in discrete amounts.

4 Some illustrative simulations

In this section we include some illustrative simulations obtained from the Excel package. The actual displays shown here were generated under Excel 2003, and minor changes to their appearance may occur under Excel 2007 or Excel 2010. It must be emphasised that static displays, as presented in this paper, by their very nature do not convey the same amount of information as do dynamic displays where physical quantities may be varied in real-time. Nevertheless it is hoped that they do give some indication of the simulations available.

4.1 Wave simulations

Figure 4 shows a "screen shot" or static display of $y(x,t)$ at an arbitrary time ($t = 88.0$ s) for two sinusoidal waves travelling in opposite directions with the same speed, the equations of the two waves being

$$y_1(x,t) = 1.0 \sin(2x - 0.2t), \tag{11}$$

and

$$y_2(x,t) = 1.0 \sin(2x + 0.2t + \pi/2). \tag{12}$$

The resultant, a standing wave, characterised by nodes (points of zero displacement at all times) and anti-nodes (points of maximum amplitude of displacement), is also shown in Figure 4. Figure 5 shows $y(t)$ at $x = 0$ for the waves of Figure 4. Note that for Figure 5, data extends only to $t = 88.0$ s, the time at which Figure 4 is shown. The $y(t)$ graph, although more difficult to generate, has proven to be particularly useful as we have found that students find it quite a challenge to distinguish between the wave motion as a function x for a fixed t and the motion as a function of t for fixed x. Also the $y(t)$

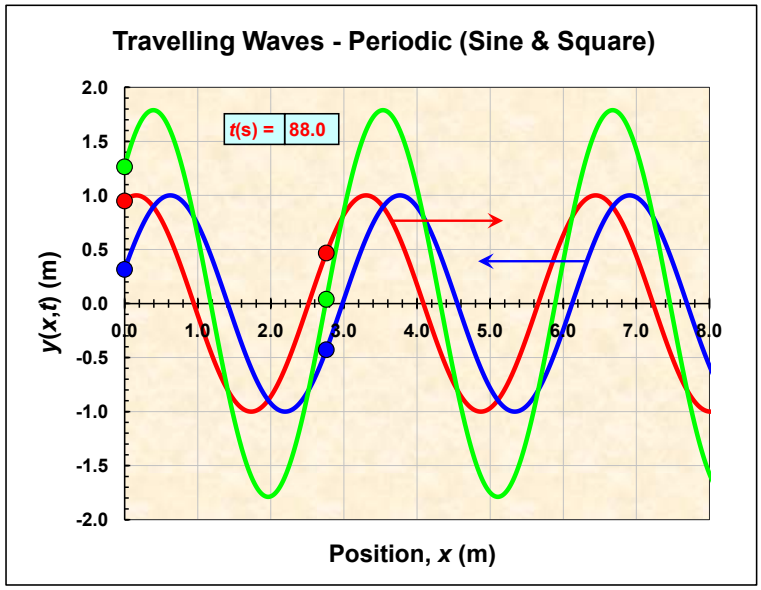

Figure 4: Illustrative results showing a static display of two sine waves of equal amplitudes ($A = 1$ m), angular wavenumbers ($k = 2$ rad/m) and angular frequencies ($\omega = 0.2$ rad/s) but different phases ($\phi_1 = 0$ rad, $\phi_2 = \pi/2$ rad) travelling with equal velocities in opposite directions, as indicated by the arrows. Their sum, a standing wave, is also shown. The filled circles at $x = 0$ and $x = 2.76$ represent markers used to highlight the motion as a function of t at fixed values of x, that at $x = 2.76$ corresponding approximately to a node in the standing wave (see text). The display is at an arbitrary time of $t = 88.0$ s, the value of t shown in the display being automatically updated as the display changes.

curve for fixed x is of help in understanding the concepts of transverse particle velocity and acceleration [see equations (4) and (5)], the former the students often confuse with the wave or propagation velocity.[1]

Figure 6 shows the transverse particle velocity, v_{T}, as a function of time at $x = 0$, appropriate to the motion of Figure 5. Being the derivative of the displacement and hence being cosine functions [see equation (4)], these curves are 90 degrees out of phase with the corresponding curves of Figure 5, as expected.

Although Figures 4, 5 and 6 are for sinusoidal waves, the display is capable of synthesizing square and triangular waves, generated by adding their Fourier components,

[1]The $y(t)$ display was more difficult to generate than the $y(x,t)$ display because a fixed length array was employed and the entire array had to be re-plotted for each time increment. This meant that array elements beyond the current time element were not to be plotted. This was accomplished by initializing the entire array to "#N/A(1)" after each time increment. Sometimes breaks occur in this display, depending at least partly on computer processing power and screen update rates. Although the $y(t)$ display works adequately, it is quite possible that it could be produced in a much cleaner manner using a macro.

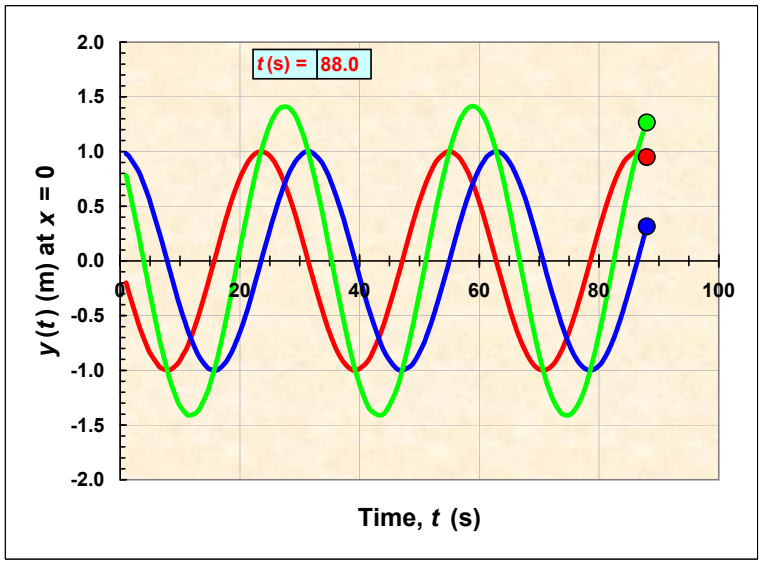

Figure 5: The motion as a function of time at $x = 0$, shown as static display, for the waves of Figure 4, up to $t = 88.0$ s (indicated by the filled circle markers), the time corresponding to that in Figure 4.

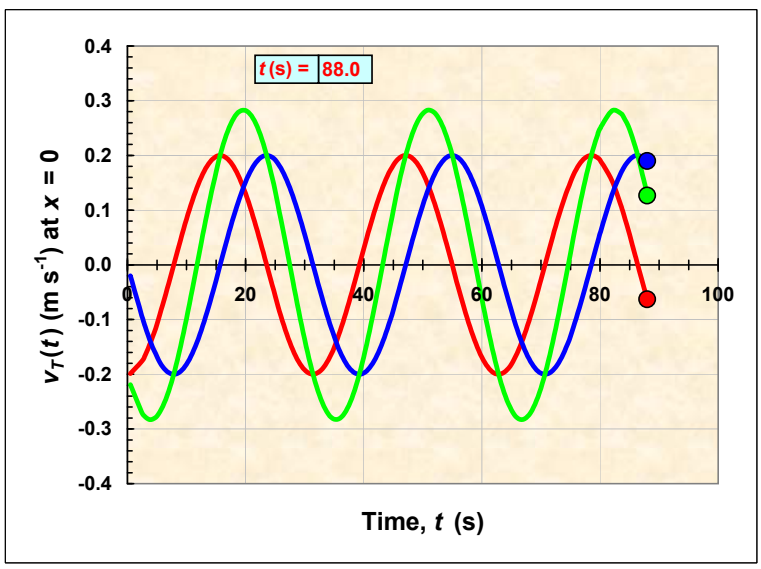

Figure 6: The transverse particle velocity v_{T}, as a function of time at $x = 0$, corresponding to the curves of Figure 5.

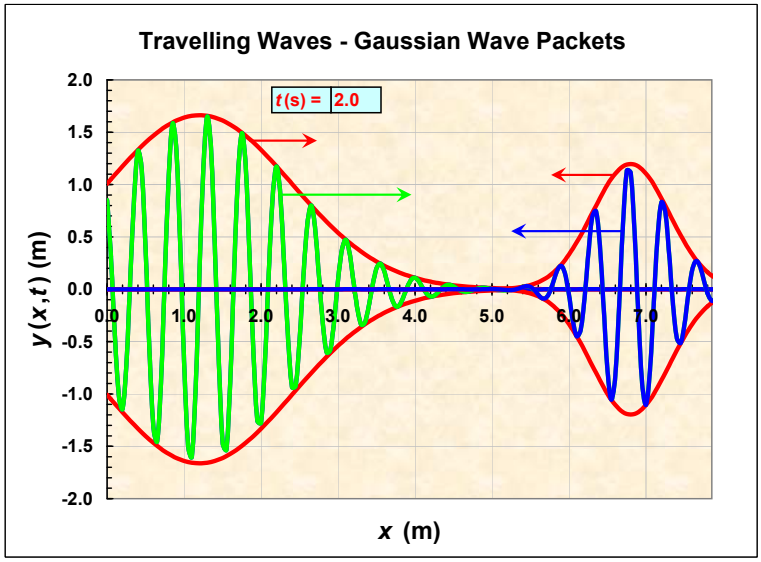

Figure 7: Two Gaussian wave packets travelling in opposite directions with the same speed and about to pass through each other. The arrows indicate the velocities of the two wave packets and the individual waves passing through each wave packet.

where the number of harmonics included may be varied (see e.g., Hecht [4]).

Figure 7 shows two Gaussian wave packets, a broad and a narrow one, travelling in opposite directions with equal speed and about to pass through each other. The relevant equations to the two packets [see equation (6)] are

$$y_1(x,t) = \frac{5\exp\left[-\frac{1}{2}\left\{(x-0.1t-1)/1.2\right\}^2\right]}{1.2\,(2\pi)^{1/2}}\cos(14x-6t)\,, \tag{13}$$

and

$$y_2(x,t) = \frac{1.5\exp\left[-\frac{1}{2}\left\{(x+0.1t-7)/0.5\right\}^2\right]}{0.5\,(2\pi)^{1/2}}\cos(14x+6t)\,. \tag{14}$$

The Gaussian envelopes each travel with a speed whose magnitude is 0.1 m/s, while the individual cosine waves pass through their respective envelopes, the magnitude of their speed being $\omega/k = 6/14 \simeq 0.429$ m/s.

Lissajous figures or curves arise when oscillations at right angles are combined, and are usually displayed on cathode ray oscilloscopes. Figure 8 shows a Lissajous figure, formed by the two waves

$$y(x,t) = 1.8\sin(8x-0.6t)\,, \tag{15}$$

and

$$x(y,t) = 1.8\sin(2y-0.2t+\pi/2)\,. \tag{16}$$

Note that for this case $k_y/k_x = 8/2 = 4$, and in the (improvised) implementation employed here it is in fact the angular wavenumber k, (sometimes referred to as a "spatial

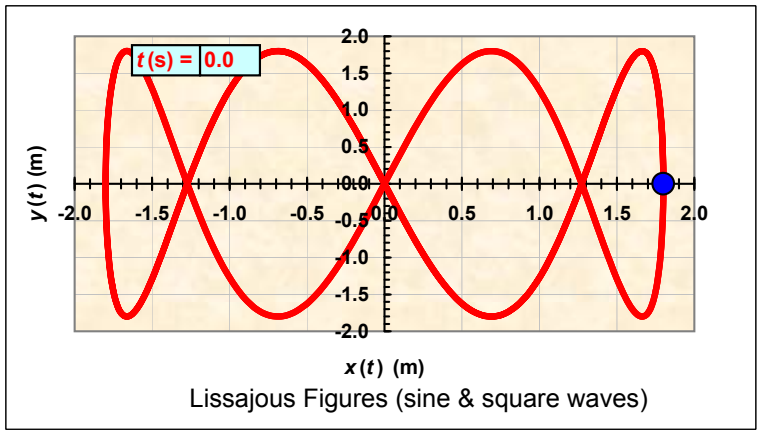

Figure 8: Example of a Lissajous figure. For the case shown, $k_y/k_x = 4$, where k_y and k_x are the angular wavenumbers of the constituent waves (not shown here) in the y and x directions respectively. The filled circle is a marker which traces out the curve and represents the current position. This figure is actually for two sine waves, but square waves, up to the 29th harmonic, may be displayed.

frequency"), rather than the angular frequency ω, which plays the primary role in determining the exact shape of the Lissajous figure formed. Interested readers are referred to the excellent Lissajous figure simulation of Wischniewsky [8], which documents mathematical details not included in the present work.

4.2 Fraunhofer diffraction simulations

Figure 9 shows the Fraunhofer diffraction pattern for two slits of finite width [see equation (7] or (9)), the well-known $\cos^2$ fringes of Young's double-slit experiment being modulated by the diffraction envelope. For the case shown $d/a = 3$, and the 3rd, 6th, 9th etc. orders of the interference pattern are "missing", being "suppressed" by the diffraction envelope.

Note that $d/a = 1$ means that the slits just touch and the pattern becomes that of a single-slit of width, in general, Na. This can be demonstrated using the parameter values of Figure 9 and allowing either d to decrease or a to increase until $d/a = 1$. It should be stressed, however, that some parameter values possible with the simulation are not physically reasonable. For example it is possible to set $d < a$ or $N < 1$, leading to unreasonable results (N is, however, restricted to integral values, as noted in §3.2).

Figure 10 shows an identical situation to Figure 9, but with N increased from 2 to 4. (In the Excel package this is accomplished by either manually changing N from 2 to 4, or better, by allowing the time variable to increase N automatically in real-time.) The interference maxima are sharpened (and are now called principal maxima) and in between adjacent principal maxima there appear $N - 2 = 2$ weak secondary maxima. However, it may be seen that there are no other changes to the pattern—the diffraction

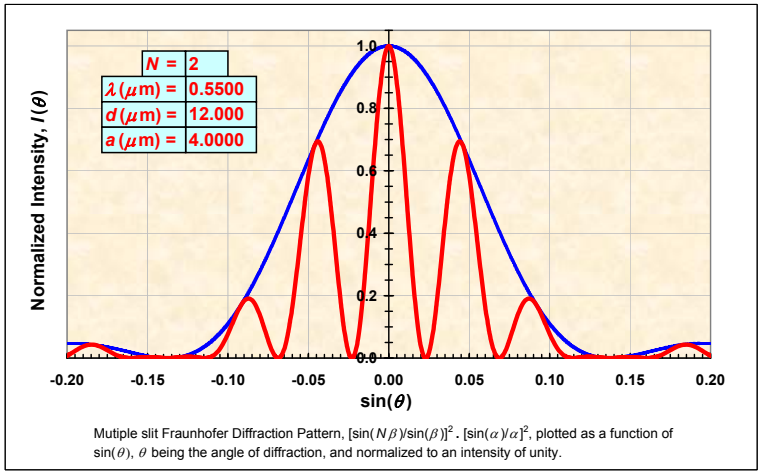

Figure 9: Example of a Fraunhofer diffraction pattern for two slits (i.e., $N = 2$) with $\lambda = 0.55$ μm, $d = 12$ μm and $a = 4$ μm. Hence $d/a = 3$ and the missing orders are therefore the 3rd, 6th, 9th etc. Note that the values of N, λ, d and a shown in the display are automatically updated as the display changes.

envelope (governed by the slit width, a) remains unchanged and the positions of the principal maxima (governed by the slit separation, d) also remain unchanged.

Figure 11 shows an identical display to Figure 10 but with d/a increased from 3 to 4 by the increase of the slit separation d, from 12 μm to 16 μm. Note that *increasing* d *decreases* the separation between the principal maxima, resulting in this case in the suppression of the 4th, 8th, 12th etc. principal maxima. This illustrates the general rule that *increasing* "something" in "object space" *decreases* "something" in "diffraction space". (In the EXCEL package, as noted above for N, increasing d may be accomplished by either manually changing d from 12 to 16 or allowing the time variable to increase d automatically in real-time.)

Figure 12 shows a similar display to Figure 11 but with N increased to 20 and for three discrete wavelengths, $\lambda = 0.45$, 0.55 and 0.65 μm. It may be seen that:

(i) The secondary maxima become of less importance as N is increased, becoming essentially of zero intensity for N very large. Also, when N becomes very large the principal maxima become very narrow and sharp, essentially becoming a spectral line if the source is monochromatic. This is the basis of the diffraction grating.

(ii) The interference pattern for each wavelength is modulated by its own diffraction envelope, increasing in width with increase of wavelength.

(iii) There is the possibility of over-lapping orders. Over-lapping orders result in the spectrum in one order being corrupted by the spectrum from another order, thereby causing an irretrievable loss of information. This is more likely to occur in the higher orders and is obviously undesirable.

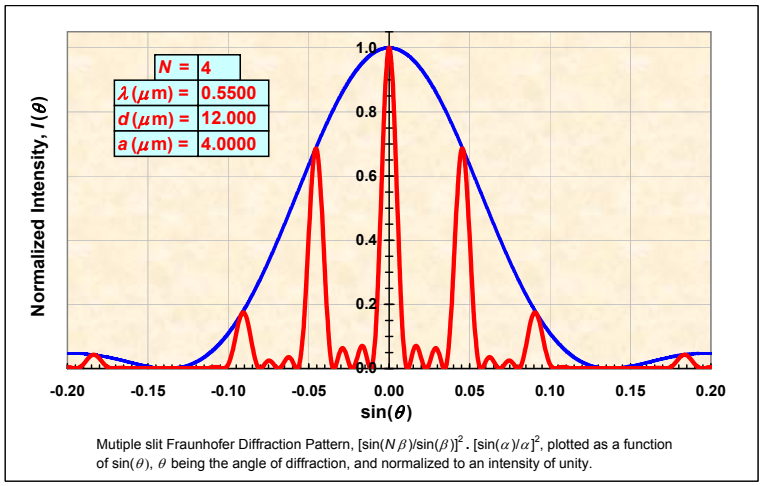

Figure 10: As for Figure 9 (i.e., $\lambda = 0.55$ μm, $d = 12$ μm, $a = 4$ μm and hence $d/a = 3$) but with N increased from 2 to 4, showing i) the sharpening of the principal maxima, and ii) the appearance between adjacent principal maxima of $N - 2 = 2$ weak secondary maxima.

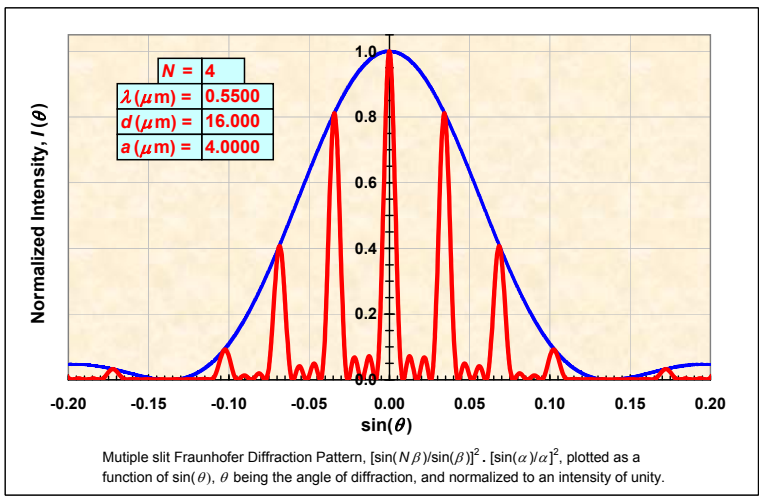

Figure 11: As for Figure 10 (i.e., $N = 4$, $\lambda = 0.55$ μm and $a = 4$ μm) but with d increased from 12 μm to 16 μm resulting in d/a increasing from 3 to 4. The 4th, 8th, 12th etc. principal maxima are now suppressed (by the diffraction envelope).

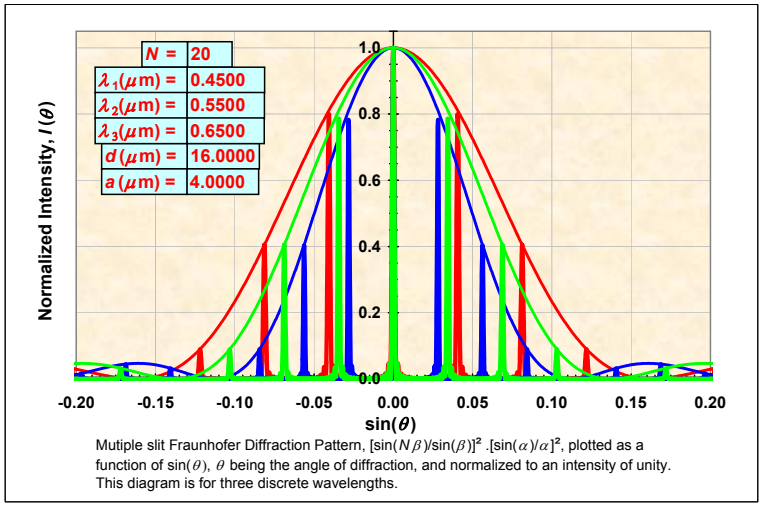

Figure 12: As for Figure 11 (i.e., $d = 16$ μm and $a = 4$ μm, and hence $d/a = 4$) but with N increased to 20. In addition three discrete wavelengths were used, $\lambda = 0.45$, 0.55 and 0.65 μm. Note that i) each interference pattern is modulated by its own diffraction envelope, and ii) that so-called "over-lapping orders" almost occurs; the third order blue line almost overlaps the second order red line. If this were to occur the second order spectrum would be contaminated or corrupted (as in fact would all higher orders).

Thus, increasing N to large enough values results in a spectrum of the source being produced in each order. A finer point regarding Figure 12 is that the central image (the principal maximum for $m = 0$, i.e., for $\sin\theta = 0$) should be uncoloured, since all wavelengths undergo constructive interference at this point generating white light. A limitation of the present display is that the colour actually shown for the central image is that of the last wavelength plotted (0.55 μm in this case).

5 Experiences in the classroom

One of us (GR) has presented a course in "Wave Motion and Optics" in first year physics to engineering students for many years. Typically the class size is more than 100 students and so it is difficult to generate personal interaction. The wave motion and optics course is usually presented as the second topic in the course, following on from the mechanics section. The students generally find the mechanics relatively easy, surviving largely on the basis of their previous knowledge of physics acquired in the last year of secondary school. The wave motion and optics section represents a quantum jump in difficulty for them, partly because the level of mathematics is higher, involving partial derivatives in the waves section, and partly because it is new material!

The waves section and the physical (as opposed to geometrical) optics section they find the most difficult. In 2005 some of the demonstrations described here were first shown in class to the students and it caused a dramatic increase in their interest in the

subject as a whole. Evidence for this was provided by the following: i) they immediately paid attention to the demonstrations (they woke up!), ii) the fact that a significant number of students requested copies of the simulations after the lectures (the demonstrations have not yet been made available on the web, although they have been provided to those who specifically requested them), iii) a number of students suggested other simulations that could be incorporated, and iv) one of the students (an Officer in the Australian Army) offered to improve the original macro controlling the timing, which at that time stepped in 1 second time intervals. Presentation of the simulations in subsequent years has produced similar enthusiastic reactions.

The demonstrations involving two waves passing through each other and the generation of their resultant disturbance were particularly useful, and appeared to help the students significantly in their understanding of the principal of linear superposition. However, notwithstanding the usefulness of the wave simulations, the Fraunhofer diffraction pattern simulations probably attracted the most interest as these enabled the students to see clearly how the various parameters in "object space" affected the various characteristics of the diffraction pattern in "Fourier transform" or "diffraction space". For example the slit width a, influences the diffraction envelope, but not the position of the interference maxima, the slit separation d, influences the positions of the interference maxima but not the diffraction envelope, and the wavelength λ, influences the extent of the entire diffraction pattern.

So in summary it appears that the simulations were useful in significantly raising the students' level of interest and also in helping their physical understanding of the subject material.

6 Summary and conclusions

In this paper we have presented an Excel package suitable for displaying physical quantities which vary in time, by use of a macro that interrogates the system clock. Being written in Excel it should be useful to those without access to more specialized packages such as Matlab. The package is well suited to displaying physical phenomena often encountered in first year physics courses in particular, for example wave phenomena in general and Fraunhofer diffraction patterns. However, other potential users, particularly those interested in real-time simulations, may find additional applications in which the time parameter may be employed directly or used to control the variation of other parameters. An obvious example is simple harmonic motion, including damped simple harmonic motion and forced oscillations. One could allow the damping to vary with time, or the frequency and amplitude of an external driving force to vary with time thereby illustrating the conditions for resonance.

The package has been used to demonstrate wave motion and Fraunhofer diffraction patterns to first year physics students for a period of about seven years and, as judged by their reactions in class and also their requests for copies of the package, it has contributed significantly to their interest and hence understanding of these phenomena.

Acknowledgements

Real time motion was originally obtained by the use of the macro "Active-Clock" written by Aaron T. Blood. This was modified and extended by Mathew Singers to allow time increments of 0.1 seconds. We acknowledge both of their contributions as it goes without saying that without these macros real-time displays would not have been possible. We also thank John Cole for useful discussions on some of the more subtle aspects of Excel and Annabelle Boag for assistance with Figure 1. Finally we thank three anonymous referees, both for drawing to our attention a number of papers relevant to this work and for their constructive suggestions, which we think have led to the significant improvement of this paper.

Appendix

Below is an extract from the VBA macro code used to facilitate real-time simulations. The subroutine “stopwatch” shown below is assigned to the “Start” CommandButton (see §3.2). In addition the subroutines “myStop”, “myReSet” and “stopwatchreverse” are assigned to the “Stop”, “Reset” and “Reverse” CommandButtons respectively. (See the full listing of the VBA macro code for these latter three subroutines.)

```
Sub stopwatch()
    'Seconds and fractions of seconds Timer (accuracy: 0.1 seconds).
    '
    'Subroutine "stopwatch" is assigned to "Start" CommandButton.
    '
    Dim Start, Finish, TotalTime
    '
    'Set timer.  Write current time into "Start" to facilitate calculation
    'of total time. The "Timer" function returns the number of seconds
    'elapsed since midnight, to an accuracy of 0.1 seconds.
    Start = Timer
    '
    'Set "StopSW" and "ReSetSW" to "False".
    '
    StopSW = False
    ReSetSW = False
    '
    'Times will be written into cell "f4".
    'Write current value of contents of cell "f4" into "myTime".
    myTime = Range("f4").Value
    Range("f4").Select
    '
myStart:
    This is the start of a "Loop" and starts the time running in the
    'positive time direction.
    '
    'Yield to other processes.
    DoEvents
    '
    'Calculate time.
    'Write current time into "Finish" in order to calculate total time since
    '"Start" pressed.
    Finish = Timer
    TotalTime = Finish - Start
    '
    'The "DoEvents" immediately below was critical to the successful running
    'under Excel 2007.
    DoEvents
    '
    'Add "TotalTime" (time elapsed since cell "f4" last updated) to "myTime"
    '(current contents of cell "f4"), and show time on sheet.
```

```
    Range("f4").Value = Format(myTime + TotalTime, "0.0")
    '
    'Test for "ReSet" and if "True" set cell "f4" to "0", clear cell "g4"
    'and set "StopSW" to "True".
    If ReSetSW = True Then
        DoEvents
        Range("f4").Value = 0
        Range("g4").Value = ""
        StopSW = True
    End If
    '
    'Test for "Stop".  If "False" write "Forward" into cell "g4", to indicate
    'time is running in the forward direction, and loop back to "myStart".
    'If "True", clear cell "g4" and quit.
    If StopSW = False Then
        DoEvents
        Range("g4").Value = "Forward"
        'Go back to myStart if neither "Reset" nor "Stop" has been pressed.
        GoTo myStart
    Else
        Range("g4").Value = ""
    End If
    End
End Sub
```

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