Spreadsheets in Education (eJSiE)

Volume 4 | Issue 2 Article 5

10-17-2010

Radiometric Dating for Novice Learners: Visualizing, Modeling, and Simulating via Animated Spreadsheets

Scott A. Sinex
Prince George's Community College, ssinex@pgcc.edu

Follow this and additional works at: http://epublications.bond.edu.au/ejsie

Recommended Citation
Sinex, Scott A. (2010) "Radiometric Dating for Novice Learners: Visualizing, Modeling, and Simulating via Animated Spreadsheets," *Spreadsheets in Education (eJSiE)*: Vol. 4: Iss. 2, Article 5.
Available at: http://epublications.bond.edu.au/ejsie/vol4/iss2/5

This In the Classroom Article is brought to you by the Faculty of Business at ePublications@bond. It has been accepted for inclusion in Spreadsheets in Education (eJSiE) by an authorized administrator of ePublications@bond. For more information, please contact Bond University's Repository Coordinator.

Radiometric Dating for Novice Learners: Visualizing, Modeling, and Simulating via Animated Spreadsheets

Abstract
Radiometric dating is introduced to beginning students in a real-world fashion through animated spreadsheets. Students discover the behavior of the parent-daughter relationship in radioactive decay, how the parent-daughter ratio yields the age, and a number of assumptions involved in radiometric dating. After exploring the concepts, and with some elementary mathematical modeling skills, students analyze data from the literature.

Keywords
radioactivity, radiometric dating, discovery learning, modeling and simulation

Cover Page Footnote
The author wishes to thank Barbara Gage from Prince George's Community College and the two anonymous reviewers for helpful comments that improved the manuscript.

This in the classroom article is available in Spreadsheets in Education (eJSiE): http://epublications.bond.edu.au/ejsie/vol4/iss2/5

1. Introduction

Radioactive decay and its use in the determination of the age of an object [4] are encountered in beginning college Earth science and chemistry. Using only college algebra, it is possible to bring radiometric dating to the level used by geochemists and provide a more dynamic conceptual picture via animated spreadsheets. Most introductory textbooks oversimplify the process. This article shows how data for concordia and isochron plots are analyzed and how a variety of factors can influence the calculated age. Actual geological ages are calculated and some good mathematical modeling principles applied to evaluate the results. Students need some elementary mathematical modeling background [8], a really good science process for them to know (and classically lacking in introductory textbooks) to use these spreadsheets.

2. The Radioactive Decay Concept

Radioactive decay is a classic first-order kinetics process that can be summarized by the decay reaction below in eq 1 and illustrated via animated multiple representations in Figure 1 (*concept tab* from the age_dating.xls file):

$$\text{Parent } (P) \longrightarrow \text{Daughter } (D) + \text{decay emission} \qquad (1)$$

The initial amount of parent, P_o, is set at the time of crystallization of magma (molten rock material) to form an igneous rock. It is assumed that the rock remains a closed system (no loss or gain of P or D by any other processes) as time passes. Figure 1 taken from the *concept tab* demonstrates the macroscopic-microscopic connection. It would be to the reader's advantage to open the accompanying spreadsheets to follow along.

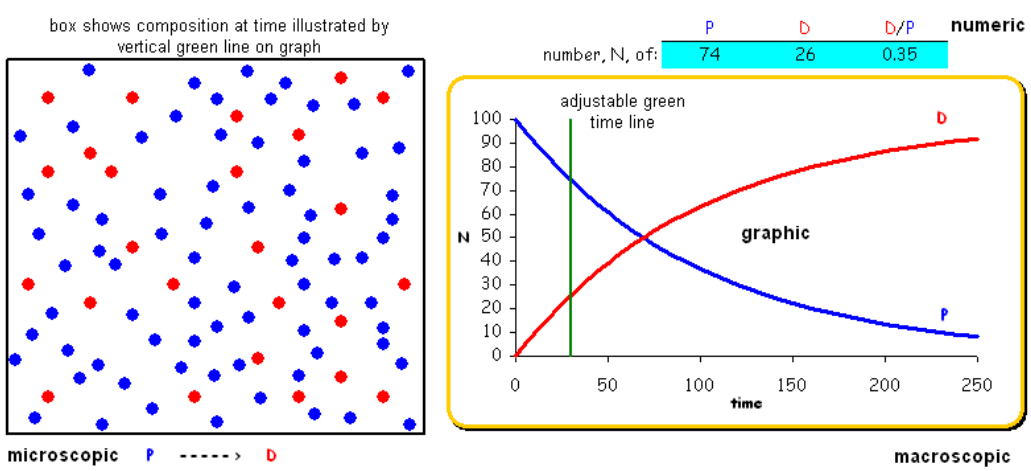


Figure 1: Parent-Daughter Relationship (*concept tab*) with multiple representations

The parent decreases in an exponential fashion while the daughter experiences limited exponential growth. The integrated rate law or equation (symbolic) is given by eq 2:

$$P = P_o e^{-kt} \tag{2}$$

where P is the amount of parent at time = t, P_o is the initial amount of parent at time = 0, and k is the rate constant of the nuclear decay (usually referred to as the decay constant, λ, by nuclear chemists). On a simple chemical basis, $P_o = P + D$, so if one can measure P and D and know k, the age, t, can be calculated. Now this assumes that all the D in the sample was produced from the decay of P (or the initial amount of daughter, $D_o = 0$). This is the typical presentation in most introductory textbooks [5]. Via mass spectrometric measurements the D/P ratio can be very accurately determined, so the eq 2 above becomes eq 3:

$$\frac{P}{P_o} = \frac{P}{P+D} = e^{-kt} \tag{3}$$

On inverting and some simplification (eq 4):

$$\frac{P+D}{P} = 1 + \frac{D}{P} = e^{kt} \tag{4}$$

Eq 4 shows that the D/P ratio is a function of time or age and that it depends on the rate constant for the parent in question. We will examine this equation in the next section after considering half-life.

In the discussion of radioactive decay it is customary to use the half-life. Half-life, $t_{1/2}$, is the amount of time required for P_o to go to $P_o/2$ or more generally the time interval for any P to go to $P/2$. Using eq 2 we can relate the half-life to the rate constant as seen in eq 5.

$$t_{1/2} = \frac{0.693}{k} \tag{5}$$

For most geological dating, long-lived radioisotopes are used. Table 1 lists some common radioisotopes used in the Earth sciences to date materials older then ten million years old (the K-Ar method can date to as young as tens of thousands of years old [4]).

Table 1: Long-lived Radioisotopes [10]

Parent	Daughter	Mode of Decay	Rate Constant (decay constant)	Half-life	Stable Isotope**
^{40}K	^{40}Ar	electron capture	5.543×10^{-10} yr	1.28×10^{9} yr	^{36}Ar
^{87}Rb	^{87}Sr	beta decay	1.42×10^{-11} yr	4.8×10^{10} yr	^{86}Sr
^{147}Sm	^{143}Nd	alpha decay	6.54×10^{-12} yr	1.06×10^{11} yr	^{144}Nd
^{232}Th	^{208}Pb	6 alpha, 4 beta*	4.948×10^{-11} yr	1.4×10^{10} yr	^{204}Pb
^{235}U	^{207}Pb	7 alpha, 4 beta*	9.849×10^{-10} yr	7.07×10^{8} yr	^{204}Pb
^{238}U	^{206}Pb	8 alpha, 6 beta*	1.551×10^{-10} yr	4.47×10^{9} yr	^{204}Pb

* first steps for each are alpha (slow step controls half-life), remaining steps are fast
**non-radiogenic isotope (not produced by radioactive decay)

3. Using the Daughter-Parent Ratio (Assumes $D_o = 0$)

How does the daughter-parent ratio, *D/P*, behave over time? The *P to D decay tab* illustrates how the *D/P* ratio grows as time increases and allows the half-life to be adjusted. Let's consider a plot of *D/P* as a function of time (eq 6) and its behavior. If the age, *t*, is less than half-life, $t_{1/2}$, the plot is near perfectly linear (very high r^2 value for linear regression) and obviously begins to curve as *t* approaches $t_{1/2}$.

$$\frac{D}{P} = e^{kt} - 1 \approx kt \text{ (when } kt << 1) \tag{6}$$

The *DP ratio tab* (Figure 2) animates this equation for a number of *P-D* pairs over a range of time scales. The error from the *kt* approximation is also shown and will only be seen as *t* approaches $t_{1/2}$. The approximation, eq 7, comes from a Taylor series expansion of the exponential function [10].

$$e^{kt} = 1 + kt + \frac{(kt)^2}{2!} + \frac{(kt)^3}{3!} + \ldots \tag{7}$$

where the higher order terms are small when kt << 1

From a measurement of the *D/P* ratio the age, *t*, of the material can be determined. Does this really work?

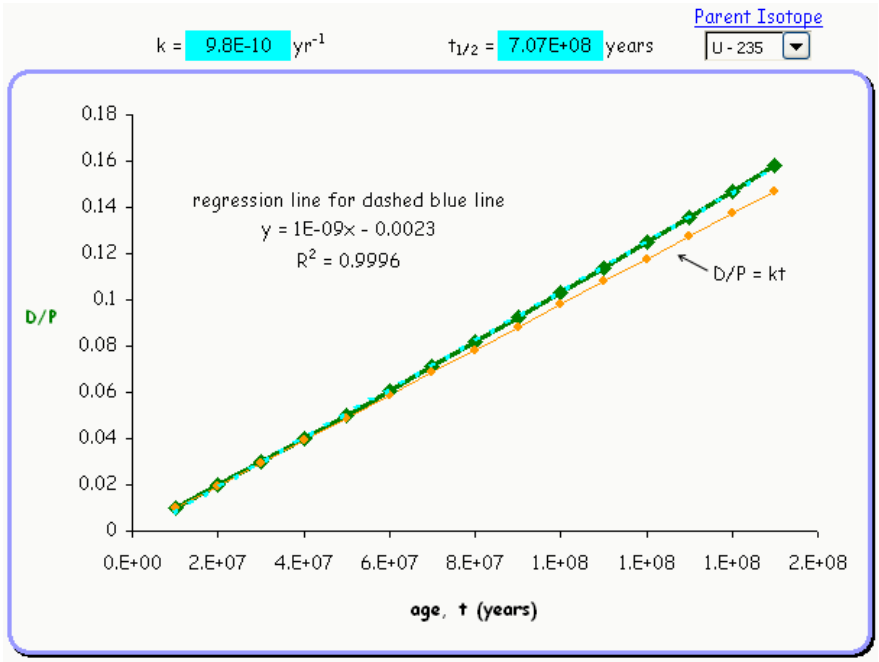


Figure 2: D/P Ratio over Time (*DP ratio tab*)

Now for ^{232}Th, ^{235}U, and ^{238}U and their lead daughter products, the parents are found in minerals such as zircon ($ZrSiO_4$), which contain very little lead on crystallization. Hence the *D/P* ratio can be used to date zircons since D_o is essentially zero. Since we have two U-Pb pairs to work with they can be used to check the method as both ratios should give the same age (this assumes no lead loss). The Th-Pb pair would add a third check as well. The equations (eq 8) for the all three pairs are given below and the U-Pb pairs illustrated in Figure 3. A concordia diagram, which shows the U-Pb pairs plot as a line if both ratios yield the same age, is shown as lower part of Figure 3.

$$\frac{^{207}Pb}{^{235}U} = e^{k_{235}t} - 1$$

$$\frac{^{206}Pb}{^{238}U} = e^{k_{238}t} - 1$$

$$\frac{^{208}Pb}{^{232}Th} = e^{k_{232}t} - 1 \qquad (8)$$

The three parents above are not simple parent-daughter decays; they are multi-step decays or decay chains [10] as seen in eq 9.

$$P \xrightarrow{\alpha} \ldots \longrightarrow \ldots \longrightarrow D \qquad (9)$$

The slowest step, which is an alpha decay for the first step of all three parents, controls the rate of decay. These systems are in equilibrium in 10 million years time. This equilibrium, referred to as secular equilibrium in nuclear chemistry, is obtained due to the very long parent half-life compared to the short daughter half-lives in the chain. These chain decay systems approach secular equilibrium in about 7-10 half-lives of the longest daughter half-life in the chain. For details of secular equilibrium, really a steady state, see Zhang [11].

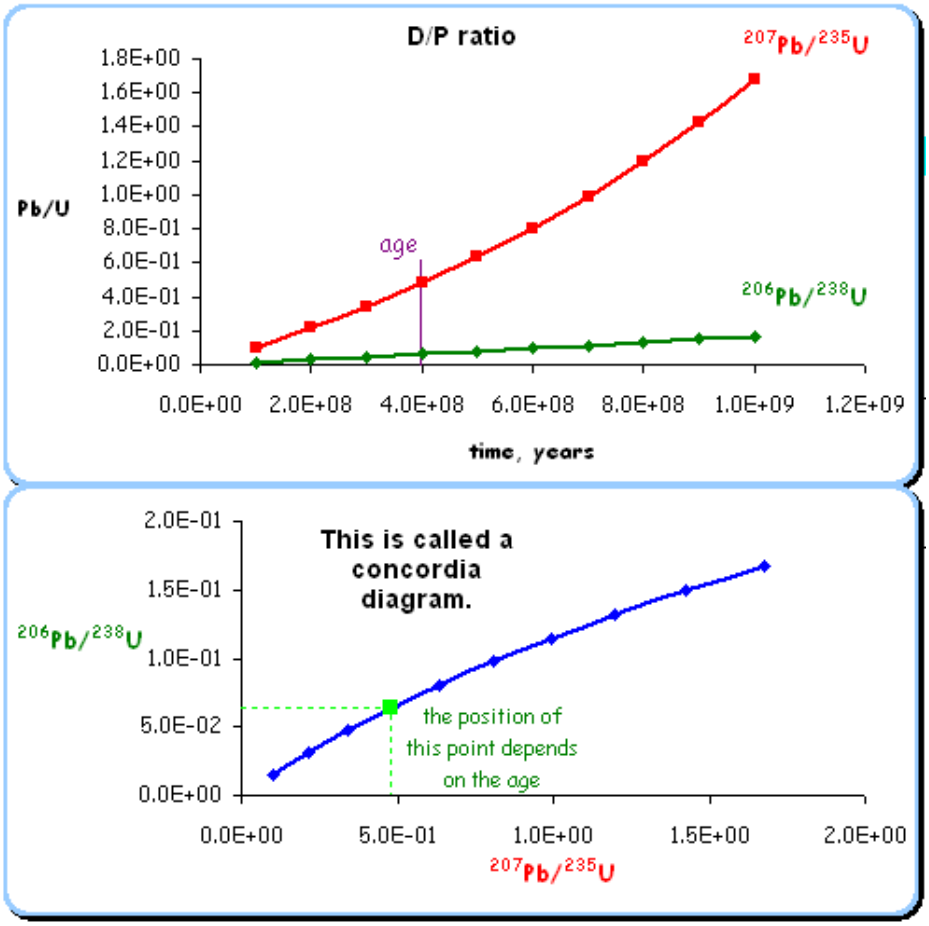


Figure 3: U-Pb Dating in Zircons (*zircon tab*) with tracer line for age

Does this work? Table 2 gives the concordant results (within experimental uncertainty) for a number of minerals using all three pairs in most cases. Hence these minerals must have remained as closed systems and our assumption that $D_o = 0$ held true.

Table 2: Isotope ages in million years [2]

Mineral	Location	$^{206}Pb/^{238}U$	$^{207}Pb/^{235}U$	$^{208}Pb/^{232}Th$
samarkite	Spruce Pine, NC, USA	314	316	302
zircon	Witchita Mtns, OK, USA	520	527	506
zircon	Ceylon	540	544	538
pitchblende	Katanga, Belgian Congo	575	595	--
uraninite	Romteland, Norway	890	892	900
thucolite	Witwatersrand, South Africa	2110	2080	--

What happens to the *D/P* ratio if the initial daughter, D_o, is not zero? On the *DP ratio tab*, students can investigate and discover the error in age by increasing D_o. Students should observe a high positive error in the age (apparent age > actual age). When $D_o > 0$ a systematic error (constant positive bias) occurs in the age. With $D_o > 0$ can we still determine a correct age for a sample? This leads us into the next section.

4. The Isochron Plot ($D_o > 0$)

We can take the *D/P* equation given earlier (eq 6), rearrange it, and then add a term that considers the amount of daughter in the sample before the parent contributes daughter. This is the initial daughter, D_o. Since it is more accurate to determine a ratio or relative abundance by mass spectrometry, all terms are divided by *S*, a non-radiogenic nuclide (see Table 1).

$$\frac{D}{P} = e^{kt} - 1$$

rearranging

$$D = P(e^{kt} - 1)$$

and if $D_o > 0$ *then*

$$D = D_o + P(e^{kt} - 1)$$

ratio to stable isotope

$$\frac{D}{S} = \frac{D_o}{S} + \frac{P}{S}(e^{kt} - 1)$$

rearrange again and remember $y = mx + b$

$$\frac{D}{S} = (e^{kt} - 1)\frac{P}{S} + \frac{D_o}{S} \qquad (10)$$

Hence, a plot of *D/S* as a function of *P/S* (eq 10) will be linear with a slope of (e^{kt}-1) and a y-intercept of D_o/S. Since the line is for equal times, this plot is called an isochron plot. The age, *t*, is found from the slope of the line as seen in eq 11 (the uncertainty of the age is discussed in section 5).

$$t = \frac{ln\ (slope + 1)}{k} \tag{11}$$

Figure 4 illustrates a screenshot of the animated ^{87}Sr - ^{87}Rb isochron (*isochron tab*). When the original rock crystallizes, the isochron is a horizontal line (slope = 0). As time passes the line increases in slope. The *isochron II tab* gives a generic isochron where the P-D pair can be varied.

In using the isochron plot equation, it is assumed that the system was at isotopic equilibrium (homogeneous uniform value of D_o) at $t = 0$ and that the system is closed over time [10]. If this was 100% true and there was no analytical measurement uncertainty, then an isochron plot should have an r^2 value of one! The degree to which these two assumptions are violated is what adds scatter to the data (lower r^2 values) and adds uncertainty in the age determination. This is where dealing with data for an isochron plot brings in linear regression and goodness-of-fit. This is where students need some experience with modeling linear systems [6]. The *assess* and *assess II tab*s on the spreadsheet give two real-world examples of data for students to analyze.

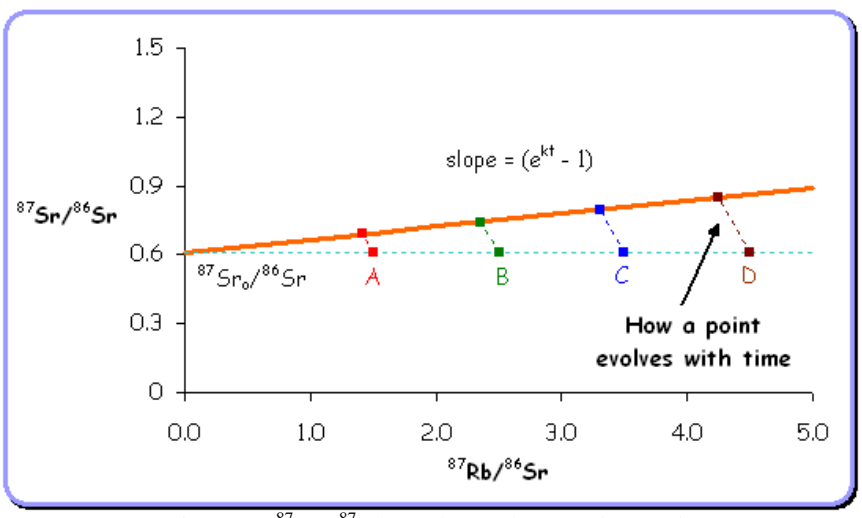
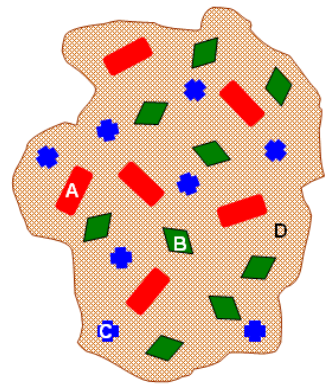


Figure 4: ^{87}Sr - ^{87}Rb Isochron Plot (*isochron tab*)

Where do the multiple points on the isochron come from? For a typical coarse-grained igneous rock such as shown in Figure 5, the minerals can be separated and analyzed. The difference in mineral structures and their chemistries are what cause the variation in parent (*P*) content of the various minerals. For a fine-grained igneous rock such as volcanic ash layers, multiple samples of the rock are analyzed. Here small differences in cooling rate cause small heterogeneities in the rock as it cooled from a more homogeneous magma.



Coarse-grained igneous rock with three large minerals (A, B, C) and finer crystalline matrix (D)

Figure 5: Sample for Dating

A good student assessment question for the isochron plot (Figure 4) is to explain the path direction (dashed line) and to ask why the path for D is longer than the path for A. This is due to the higher level of radioactive isotope (a concentration effect).

5. Factors that Influence Ages

Getting into the factors that influence the age is another level higher and uses the age_dating_2.xls file. To start we examine the "setting of the clock" with $D_o = 0$ and $D_o > 0$ in a conceptual fashion on the *setting the clock tab*. Then a "just-add-data" worksheet on the *isochron error tab* allows the errors in the slope and y-intercept to be calculated [4] and place an uncertainty on the age of the sample (eq 11). We also examine the uncertainty caused by the rate constant, k, on the *decay constant error tab*. From Table 1, most of the rate constants, k, have an uncertainty of less than 1 part per 100. We then explore how a datum point wanders if the system becomes open (both a gain and loss of *P* and/or *D*) on the *open system tab*. This can be a major cause of scatter in the data on an isochron plot. The *lunar rock tab* gives an example of a really good set of isochron data (more data sets to evaluate are available in [1]).

6. Student Assessment

Our students have previous experience with Excel for analyzing data and with the use of interactive Excel spreadsheets or Excelets both in and out of class. Students like the interactive nature of the spreadsheets and the visual aspects for enhancing concept understanding [7, 9]. An early version of the age_dating.xls spreadsheet was given to 33 second-semester general chemistry students as a take-home section of an exam at the end of the semester. The questions for the task are listed below.

► How does the daughter/parent or *D/P* ratio change as time passes?

► Radiometric dating methods assume that the initial amount of parent isotope, P_o, is set at the formation of an igneous rock when it crystallizes, turns solid, from magma (molten rock). We assume that the amount of daughter isotope is zero ($D_o = 0$). How is the age of the rock influenced if $D_o > 0$ from other sources?

► To correct for an initial amount of a daughter isotope (usually not known), geologists use the isochron plot. For any parent-daughter pair, how does the age, t, influence this plot?

► How does the D_o influence the isochron plot as well?

► Determine the age of the rocks for the data given on the assess and assess II tabs. Attach the graphs of the isochron plots for both, any calculations, and answers to questions on the tabs.

The results were assessed on a 30 point scale and are presented in Figure 6. The average score was 22.4 with a standard deviation of 5.9. The two low scores were students who had no idea how to calculate the age; even through the information was right in front of them on the spreadsheet.

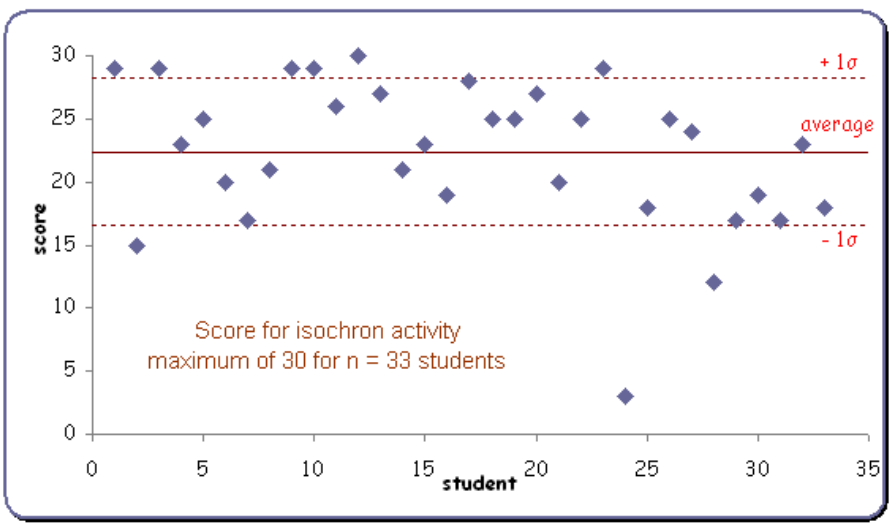


Figure 6: Assessment Results

The question "For any parent-daughter pair, how does the age, t, influence this plot?" was missed by some students, because the y-axis rescaling (autoscaling needed for various *P-D* pairs) was missed. However, the cell with the slope giving the numerical value was changing! This has been cleared up by adding the Sr-Rb isochron with a fixed y-axis.

7. Some Final Thoughts

These interactive Excel spreadsheets or Excelets provide a wealth of animated behavior for the mathematical relationships used in radiometric dating. They allow a more real-world approach to radiometric dating than is found in many introductory textbooks. Students use mathematical modeling skills to analyze real-world data.

Recently, Davila and Talanquer [3] demonstrated that a number of general chemistry textbooks lack higher-order thinking questions and problems. This article shows that through the use of animated interactive spreadsheets, instructors can add higher-order thinking questions, enhance both scientific and algebraic thinking, and help students develop a deeper conceptual understanding.

8. References

[1] Barth, A. (2009) Calculating Rb-Sr Isochrons (has a number of data sets to evaluate) http://serc.carleton.edu/NAGTWorkshops/petrology/teaching_examples/calculating_rb-sr_isochrons.html.

[2] Brownlow, A.H. (1979) *Geochemistry*, Prentice-Hall Inc, Englewood Cliffs, NJ, 498pp.

[3] Davila, K. and Talanquer, V. (2010) Classifying End-of-Chapter Questions and Problems for Selected General Chemistry Textbooks Used in the United States, *J. Chem. Educ. 87*, 97-101.

[4] MacDougall, J.D. (2008) *Nature's Clocks: How Scientists Measure the Age of Almost Everything*, University of California Press, Berkeley, CA, 271pp.

[5] Moore, J.W, Stanitski, C.L., and Jurs, P.C. (2008) Chemistry: The Molecular Science, 3rd ed., Thomson Higher Education, Belmont, CA; Silberberg, M.S. (2010) Principles of General Chemistry, 2nd ed., McGraw Hill, New York, NY; Tro, N.J. (2011) Chemistry: A Molecular Approach, 2nd ed., Pearson Education, Upper Saddle River, NJ.

[6] Sinex, S.A. (2005) Exploring the goodness of fit in linear models, *Journal of Online Mathematics and its Applications*, vol. 5. [Online] Available: http://mathdl.maa.org/images/upload_library/4/vol5/index.html.

[7] Sinex, S.A. (2007) Chemical spreadsheet investigations: Empowering student learning of concepts via camouflaged mathematical exploration, *Using Computers in Chemical Education Newsletter*, Fall 2007 issue. [Online] Available: http://academic.pgcc.edu/~ssinex/UCCEN_F07/UCCEN_F07.pdf.

[8] Sinex, S.A. (2008) Scientific and Algebraic Thinking: Visualizing with Interactive Excel and Nested Styrofoam Cups, *On-Line Proceedings of the 34th Annual Conference of the American Mathematical Association of Two-Year Colleges* in Washington, DC.

[9] Sinex, S.A. and Halpern, J.B. (2010) Discovery learning tools in materials science: Concept visualization with dynamic and interactive spreadsheets, in *Proceedings of the Materials Research Society Symposium: Materials Education*, vol. 1233, 91-96.

[10] White, W.M. (2009) Chapter 8: Radiogenic Isotope Geochemistry online at: http://www.geo.cornell.edu/geology/classes/Chapters/Chapter08.pdf

[11] Zhang, Y. (2008) *Geochemical Kinetics*, Princeton University Press, Princeton, NJ, 656 pp.