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False Beliefs and Game Theory: Implications from the Japanese Comic Story Konnyaku Mondô[¤][y]

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Abstract

This is a dialogue between two professional people on the new …eld called "epistemic logics and game theory". One speaker is a specialist who has been working in this …eld for a long time, and the other is a game theorist who is quite new to the …eld and younger than the specialist. They start discussing the Konnyaku Mondô and …nd that it has many implications for game theory in terms of its foundations and scope.

[Setting: Jan Hummer, a lecturer from a foreign land, is visiting a well known institution in a remote research village north-east of Tokyo. Kurai Shinzuki, a prominent professor at the institute, has agreed to engage in discussions with the visitor. Acts 1 and 2 take place in a laboratory with dim lighting and inadequate heating]

Act 1: Konnyaku Mondô

Jan: Hey, Shinzuki, I heard from your graduate students that you had discussed a Japanese comic story, called something like the Cognac Melon. They claimed it is quite interesting and important for game theory. Can you tell me that story?

Shinzuki: Ha, ha. I think it must be the Konnyaku Mondô, instead of the Cognac Melon. I can, of course, explain the story to you. Didn't my graduate students give an adequate explanation?

Jan: No, they didn't. They seemed unable to explain it since they did not read the story, but only heard it from you. One of them claimed it is written in

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classical Japanese which is only understandable by some old professors. Was it written so long ago?

Shinzuki: No, no, it was written from an oral presentation only about 100 years ago.[1] It is di¢cult for young people, in fact, I confess I feel it is slightly di¢cult for me too.

Jan: Huhm..., Japanese has changed a lot in only 100 years, hasn't it? Anyway, let's have the Cognac Melon..., sorry, Konnyaku Mondô.

[Shinzuki assumes a more professorial mode of speaking]

Shinzuki: The literal translation of the Konnyaku Mondô is "Devil's Tongue Jelly Dialogue".

Jan: What? What is "devil's tongue jelly"?

Shinzuki: It is a food product, like a jelly, made from roots of the Konnyaku plant. You had it at the sake restaurant we went to last week. It is usually brown and looks like a devil's tongue.

Jan: Yes, I remember it. But, why is that funny jelly food important to game theory?

Shinzuki: The jelly food itself is not relevant to game theory, of course. The story of the Konnyaku Mondô is related to game theory in many respects. It is about false beliefs and the subjective nature of people's thoughts. It is an example of how inconsistent subjective thoughts may evolve, even with common knowledge and communication.

Jan: Is that true? It sounds incorrect, or if it is true, it has serious implications for game theory. Please explain the story to me?

Shinzuki: I am not sure I will be able to reconstruct the entire story in a manner meaningful to you. Nonetheless, I shall make an attempt.

[Shinzuki now assumes an even more professorial, but somewhat dramatic, tone of a storyteller]

"There was a temple without a monk for some period. A Konnyaku maker, named Rokubei, who had lived next to the temple, moved into the temple and started pretending to be a monk. One day, a Zen Buddhist monk visited the temple and challenged the new master of the temple to a dialogue on Buddhist thoughts. Since Rokubei had no idea about Buddhist dialogue, he refused at …rst but eventually agreed."

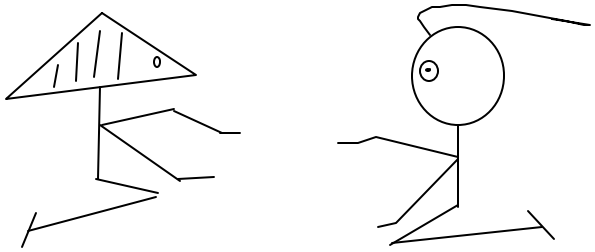
Jan: OK, we have the food producer, Robeki, who has no knowledge on Buddhist thoughts about to engage in a dialogue on Buddhist thoughts with a Buddhist monk. It looks like a sure victory. How possibly could there be any implications for game theory? By the way, are these two the only characters in the story? No beautiful princess?

[1] pp.61–70 in [16]. The Konnyaku Mondô is from the rakugo form of traditional Japanese comic storytelling. In this tradition, a story is performed by a single professional.

Shinzuki: First of all, the jelly maker's name is Rokubei, not Robeki. Secondly, you should be patient. A lot of implications will be found after our story concludes. Finally, another character is involved. His name is Hachigoro and his role is to be a witness to the entire story. I am sorry you are disappointed by …nding no love stories or battles of the sexes included here.

Let me continue the story.

"Since Rokubei did not know how to communicate with the monk on Buddhist, he did not answer the monk's questions. The monk thought that being silent might be a style of dialogue, and tried asking questions in several di¤erent fashions. After some time, Rokubei began answering the monk's questions with hand gestures. Taking this as a style of dialogue, the Monk began responding with hand gestures as well. Eventually, both Rokubei and the monk agreed that Rokubei had defeated the monk. Then the monk left as a loser."



Monk Rokubei

Jan: Why? I do not understand how Rokubei defeated the monk. What happened?

Shinzuki: I am glad to hear you …nd the story strange. Perhaps here you will …nd the implications to game theory you are searching for. As you shall see, Hachigoro both witnesses the event and plays an important role by questioning the outcome. The concluding part of the story is as follows:

"After the dialogue, Hachigoro followed the monk and asked him about the dialogue. The monk answered that the master had expressed great Buddhist thoughts through his gestures and he should be respected. Hachigoro wondered about when, if ever, Rokubei had learned Buddhist thoughts, and he returned to the temple to ask him about it. Rokubei said he had never learned Buddhist thoughts. Rather, the monk started talking badly about his jelly products with his gestures. This angered Rokubei, and thus he beat the monk."

Jan: Wait a second. I understand the monk talked to Rokubei with gestures about Buddhist thoughts. Did the gestures by Rokubei have di¤erent meanings? What are they?

Shinzuki: In fact, in the full form of the story, Hachigoro heard the meanings of the gestures intended by the monk, as well as by Rokubei. For example, a gesture of creating a circle with one's arms means the universe for the monk, but it means a round Konnyaku product for Rokubei. All the gestures are meaningful to each person.

Jan: I understand the story itself, but where are the implications for game theory?

Shinzuki: All right, now, perhaps you are ready to understand the game theoretical implications. Each of Rokubei and the monk believed that they had a perfectly meaningful dialogue, and that the victory of Rokubei was common knowledge. However, the monk believed that they had a Buddhist dialogue, while Rokubei believed that they had discussed his jelly products.

Jan: Great. I understand your explanations about the story, but I am still not able to see any implications for game theory. May I ask some questions?

[Without waiting for a response, Jan charges on]

First of all, did the gestures they exchanged become common knowledge? Secondly, is the victory of Rokubei common knowledge between them? I believe that your answer to these questions will be yes.

However, these people attached completely di¤erent meanings to their gestures. They do not share a basic understanding of what the gestures mean. In such a situation, can we say that they share the common knowledge of the victory of Rokubei as well as the gestures exchanged?

[Shinzuki becomes slightly overwhelmed]

Shinzuki: How can you ask such sharp questions to me? Let me think about them, uhum....

I would like to answer "Yes" to your …rst questions. The gestures exchanged and Rokubei's victory became common knowledge. Here, it is at least necessary to assume that their visions are normal. The speed of the light is almost in…nity, which is actually about 300,000 kilometers per second. Through their visions, they are able to verify in an instant, almost in…nite observations of a gesture and the other's observation of that gesture. Hence, the common knowledge of the gestures and of Rokubei's victory, expressed and understood with gestures, is an adequate assumption. Incidentally, I found this argument while reading the "analogy of the sun" in Chapter 6 of Plato's Republic [13].

Now more to the point, the symbolic forms of the gestures can be assumed to be common knowledge. But these symbolic forms have completely di¤erent meanings to Rokubei and the monk. Therefore, the gestures and victory are common knowledge at a super…cial level. In fact, they lead those people to profound misunderstandings of each other. Nevertheless, they still have the common knowledge of the gestures and victory.

Jan: Yes, you understand the point of my questions. When we talk about common knowledge or even about knowledge, the truthfulness is somehow as-

sumed. In our story, the common knowledge of the gestures cannot be true since the two people attached different meanings to them.

Shinzuki: Isn't the common knowledge of the gestures true? Huhm..., I think your reasoning may be correct, but your conclusion is not true. The objects of common knowledge in our case are the symbolic forms of the gestures exchanged. The common knowledge here does not include the interpretations of the gestures. May I use another analogy?

Jan: Of course, but do not digress too much.

Shinzuki: A Japanese apple farmer would like to sell his apples, which are dear to him and should be eaten fresh. An American visiting Japan wants apples to bake apple pies. Without communicating their intentions, the exchange of apples and money occurs between these two people. With communication between the two, the exchange might not have occurred. But my point here is that apples are apples independent of the intentions attached to them by the farmer and the visitor. I would like to call this the "analogy of apples" in analogy with Plato's "analogy of the sun".

Jan: Once again, you have told a nice story, but where is the relevance to our discussion?

Shinzuki: I am sorry, perhaps I should return to the truthfulness of the common knowledge of gestures. The truthfulness of the knowledge of a gesture is the accurate observation and recollection of the gesture. The intended interpretations of gestures are irrelevant as are the intentions of the buyers and sellers of apples.

Jan: Wow. You have brought out many interesting and puzzling aspects of the story. I will need some time to digest them. I should go home now and it is getting cold. Let's continue our discussion tomorrow morning, and could we start talking about game theory?

Shinzuki: Yes, but tomorrow I will have a class until 10:30. Let's meet around 10:45. I will bring another heater.

Act 2: False Beliefs and Decision Making

[In the lab, Jan has been waiting for Shinzuki, and then Shinzuki appears in a good mood carrying a heater]

Jan: How was your class?

Shinzuki: Good, of course.

Jan: Last night I thought about the story more. Now I feel it may be related to game theory. I am anxious about hearing the implications of the story for game theory. However, since you have a tendency to digress, I would like to impose a constraint on our discussions. We should directly discuss the implications to game theory and its relationship to the story. The use of analogies should be avoided.

Shinzuki: Huhm..., You are harsh, but I will try my best to play within your rules.

Let us consider a very simple situation where players 1 and 2 play the prisoner's dilemma game. Could you please draw its payo¤ matrix on the blackboard and name it Figure 0?

Jan: Sure, it is an easy task. I name the prisoner's dilemma game $g^0 = (g_1^0; g_2^0)$, since I expect you want to have other games.

	s_{21}	s_{22}
s_{11}	(5; 5)	(1; 6)
s_{12}	(6; 1)	(3; 3)¤

Figure 0: $g^0 = (g_1^0; g_2^0)$

Shinzuki: Yes, you are right. I want to give two more games now, which I will write on the blackboard.

	s_{21}	s_{22}
s_{11}	(5; 0)	(1; 0)
s_{12}	(6; 0)	(3; 3)¤

Figure 1: $g^1 = (g_1^1; g_2^1)$

	s_{21}	s_{22}
s_{11}	(0; 5)	(0; 6)
s_{12}	(0; 1)	(3; 3)¤

Figure 2: $g^2 = (g_1^2; g_2^2)$

Now, suppose that player 1 thinks that the game is given as $g^1 = (g_1^1; g_2^1)$; while player 2 thinks that the game is $g^2 = (g_1^2; g_2^2)$: Each of them thinks about the common knowledge of a di¤erent game. Although the standard game theory literature assumes the common knowledge of the game structure, it would be apparently impossible to maintain this assumption in the current context. We modify this assumption to require that each player $i = 1; 2$ believes the common knowledge of the game $g^i = (g_1^i; g_2^i)$.

Jan: What? What do you mean by "Each player i believes the common knowledge of $g^i = (g_1^i; g_2^i)$"? Perhaps, I should verify if I understand you correctly. Huhm..., it is di¢cult to understand your statement. I should paraphrase it into a longer form: "Player i personally believes that it is common knowledge that $g^i = (g_1^i; g_2^i)$ is the game to be played." Is this paraphrasing accurate?

Shinzuki: Yes, certainly.

Jan: All right. Now, it is getting clearer. The truthfulness of the common knowledge of $g^i = (g_1^i; g_2^i)$ is assumed only in the belief of player i: Is my understanding OK?

Shinzuki: You correctly understand my assumptions.

Jan: Great. Now, I should ask what kind of mathematical language you will use to represent your game theoretical argument. I would be very surprised

if you talked about such a delicate problem without having any mathematical formulation.

Shinzuki: I will talk about that problem in the mathematical language called epistemic logic. I borrow the logic language from [5] and [7].[2] Using this language, the game theoretical assumptions I uttered, and one logical conclusion derived from them are described as follows:

[Shinzuki writes on the blackboard]:

(1) $\vdash g^0; B_1C(g^1); B_2C(g^2) \Rightarrow C(\mathrm{Nash}(s_{12}; s_{22}))$:[3]

I should give some explanations about these symbolic expressions. The first symbol, $\vdash$, means the entire statement following it is provable. The left-hand side following $\vdash$, and continuing before $\Rightarrow$, consists of assumptions, or axioms. To the right of $\Rightarrow$, we find another statement which is logically concluded from the left-hand side.

Jan: Let me interpret what you wrote on the blackboard. The first symbol in the left-hand side after $\vdash$ seems to mean that the game to be played is the prisoner's dilemma game g^0. The next two statements separated by commas seem to be that: player 1 believes[4] the common knowledge of g^1, and player 2 believes the common knowledge of g^2. The right-hand side seems to mean that it is common knowledge that $(s_{12}; s_{22})$ is a Nash equilibrium. You said that the arrow means that the right-hand side is logically concluded from the left-hand side, didn't you? You also said that the symbol $\vdash$ means that the entire statement following it is provable. Isn't $\vdash$ redundant?

Shinzuki: Without the symbol $\vdash$; we only have the statement "if the left-hand side is assumed, then the right-hand side holds". The additional $\vdash$ means that the "if-then" clause is provable in an epistemic logic.[5]

Jan: OK, OK. (1) states that it is provable that if g^0; $B_1C(g^1)$ and $B_2C(g^2)$, then the strategy pair $(s_{12}; s_{22})$ as a Nash equilibrium is common knowledge. It sounds alright, but the beliefs and knowledge are mixed. In the left-hand side, the beliefs of common knowledge are assumed, while in the right-hand side, the

[2] Epistemic logic is a branch of modal logic. For a general introduction to modal logic, see Chellas [1] and Hughes-Cresswell [3]. Fagin et al [2] and Meyer-van der Hoek [11] are introductory books dealing with epistemic logics.

[3] The provability $\vdash$ depends upon a logical system. In this paper, various logical systems are involved without specifying any. For details, see Kaneko [5], Kaneko-Suzuki [7] and Kaneko et al [8].

[4] In the game theory literature, belief is often understood as subjective probability in the sense of Savage [14]. The concept of belief here differs from subjective probability in many respects. The main difference is that we discuss beliefs in association to the logical abilities of players, while beliefs are treated as a black box given together with preferences in the Savage approach.

[5] Note that it may be the case that neither the "if-then" clause nor the "if-then not" are provable, i.e., they are undecidable. This may mean that a player can find neither some conclusion nor its negation from his basic beliefs (the left-hand side). The logic approach often evaluates such provability or unprovability.

common knowledge of a Nash equilibrium is obtained. The beliefs seem to have somehow vanished and been replaced entirely by knowledge as if by magic. How did it happen?

Shinzuki: In fact, (1) is true. This is parallel to the Konnyaku Mondô in that both statements include false beliefs, but the conclusions are common knowledge. However, I admit that some unparalellism between beliefs and knowledge is involved in (1). For a better understanding, I retreat here from (1) to the following slightly weaker assertion:

(2) $\vdash g^0; B_1C(g^1); B_2C(g^2) \Rightarrow B_1C(\mathrm{Nash}(s_{12}; s_{22})) \wedge B_2C(\mathrm{Nash}(s_{12}; s_{22}))$:

We shall discuss the relationship of (1) and (2) to the Konnyaku Mondô later.

Jan: Now, (2) is nicer than (1), since each "believes" the common knowledge of something in both the left-hand and right-hand sides. Huhm..., the belief of the common knowledge of a Nash equilibrium is derived from the individual belief of the common knowledge of the game g^i. This logical calculation takes place in the mind of each player. Then what is the role of the objective statement g^0?

Shinzuki: Oh my dear, Jan, I am pleased to ...nd you understand the problem perfectly. In fact, (2) is equivalent to the following two separated statements, neither of which involves g^0:

(2a) $\vdash B_1C(g^1) \Rightarrow B_1C(\mathrm{Nash}(s_{12}; s_{22}))$:

(2b) $\vdash B_2C(g^2) \Rightarrow B_2C(\mathrm{Nash}(s_{12}; s_{22}))$:

The derivation of (2a) and (2b) from (2) needs an advanced technique of logic, while the converse is just a calculation.

Jan: It sounds wonderful, though I don't appreciate the di¢culty in the derivation from (2) to (2a) and (2b). Now on the issue of game theoretical implications, consider the decision making of player 1: He believes that his payo¤ function is g_1^1, which is the same as his payo¤ function g_1^0 of the objective game g^0. One might argue that he chooses his strategy s_{12}; because it is a dominant strategy rather than because it is part of a Nash equilibrium. Following this line of reasoning, we seem to be moving even further from common knowledge.

Shinzuki: All right, I can change (2a) into

(3a) $\vdash B_1(g^1) \Rightarrow B_1(\mathrm{Dom}_1(s_{12}))$:

Jan: I am surprised you changed your statement so easily. However, (3a) is in some sense nicer than (2a), since it includes no common knowledge. Huhm..., (3a) still includes player 2's payo¤ function in $B_1(g^1)$. Can you change (3a) into the following?

(3a$'$) $\vdash B_1(g_1^1) \Rightarrow B_1(\mathrm{Dom}_1(s_{12}))$:

Shinzuki: Yes, of course. Are you satis…ed by having a purely personalized version?

Jan: Yes, I am very satis…ed. Uhum..., it raises two opposite thoughts in my mind. On the one hand, $(3a^0)$ makes sense perfectly for me since player 1 simply derived s_{12} as a dominant strategy from the belief of his payo¤ function g_1^1. On the other hand, $(3a^0)$ no longer involves false beliefs since $g_1^1 = g_1^0$.

Shinzuki: Sorry. I change $(3a^0)$, again into

$$(3a^{00}) \ \grave{} \ B_1(g_1^2) \ ! \ B_1(Dom_1(s_{12})):$$

Please notice that I have changed the true payo¤ function $g_1^1 = g_1^0$ into the false payo¤ function g_1^2 of Figure 2.

Jan: Once again you have ‡ippantly adjusted the assumptions. I would like to understand the whole problem, instead of each small piece and it is di¢cult to concentrate when you keep changing assumptions. We seem to be moving further and further away from our initial objective. While I applaud you for refraining from the use of analogies, I must admit you are weaving an impossible web in my head with your incessant manipulation of assumptions. Please, stop and return to something concrete. I almost forget the point.

Shinzuki: Aha, I return to a full statement now:

$$(3) \ \grave{} \ g^0;B_1(g_1^2);B_1C(g_1^2;g_2^1) \ ! \ B_1(Dom_1(s_{12}))\wedge B_2C(Nash(s_{12};s_{22})):$$

Surely, we can talk about the falsity of beliefs of players in (3).

Jan: Although this looks slightly di¤erent from our starting point, it does seem to be moving back in the right direction. Let me try to make sense of (3). I start with the left-hand side of ! . Uhum..., this involves a lot of falsities. Probably, for my own sanity, I should translate (3) into English. First, it says the objective situation is described by the prisoner's dilemma game g^0. Next, it says that player 1 believes his payo¤ function is given by g_1^2, which we know to be false relative to g^0. Next, it says that player 2 believes the game $(g_1^2;g_2^1)$ is common knowledge between the players, which we know to be false both relative to the objective game g^0 and the content of player 1's belief.

In this formulation, player 1 ignores, or does not discuss, player 2's payo¤ in the formation of his beliefs, while player 2 believes they have common knowledge of $(g_1^2;g_2^1)$ which we know to be false relative to g^0. Is my understanding of the left-hand side of (3) accurate?

Shinzuki: Yes, it is.

Jan: Good. Now let's attack the right-hand side. The …rst piece is the derivation by player 1 that his strategy s_{12} is a dominant strategy. This derived statement happens to be objectively true for the game g^0. We also, …nd the derivation by player 2 that the pair $(s_{12};s_{22})$ as a Nash equilibrium is common

knowledge. The pair $(s_{12}; s_{22})$ to be a Nash equilibrium is also true relative to g^0: However, player 2 believes that this fact is common knowledge between 1 and 2, while player 1 does not think about player 2 at all. This is yet another kind of falsity

Shinzuki: Indeed, we meet a lot of falsities in beliefs, in particular, both sides of (3) involve falsities. Actually, I deliberately wrote (3) in a puzzling form.

[Jan answers in a tone of disagreement]

Jan: Yes, (3) has a lot of falsities. Huhm..., however, the falsities on both sides of the arrow ! are parallel. The previous asymmetry of falsities in (1) is, in some sense, more interesting, since the true common knowledge is derived from the false beliefs of common knowledge. As a game theorist, I would typically assume that the objective truths on the right-hand side were obtained from objective truths in the minds of the players. You have shown that this need not be the case. Is this distinction important for game theory? Can you connect this ...nding in (1) to the Konnyaku Mondô discussed yesterday?

But now my head is reeling..., and noises are rising within me.

Shinzuki: I think you are saying you are hungry. Why don't we go to the Mexican Restaurant? The lunches there are cheap and large. Although I would like to continue our discussions right after lunch, I will have another stupid meeting until 3PM. Is it convenient for you to have more discussions after that?

Jan: Yes, of course. Anyway, it is time to eat. Let's go.

Act 3: More on Decision Making

[After lunch, Shinzuki went to a meeting, and Jan has been taking a nap on a couch in the lab]

Shinzuki: Hi Jan. I am back from my hell meeting. Ah, I am sorry to wake you up.

Jan: No, no, thank you for waking me up. I wanted to wake up by myself. I was very sleepy because of the intense discussions this morning and the good food in the restaurant. How was your meeting?

Shinzuki: Nothing important as usual. Actually, I didn't sleep well in the meeting because the chairperson spoke so loudly.

Jan: Ha ha, you are in almost the same state as me. By the way, I thought about our discussions during my nap. Your explanations diverged quite a lot.

Shinzuki: I have a more general statement including the previous ones, which I will write on the blackboard as:

(4) ` ¡^{0};B_1(¡$_1$);B_1(¡$_2$) ! $B_1(D_1(s_1))$^$B_2(D_2(s_2))$:

Here, the ...rst assumption set ¡^{0} is the objective description of the situation, and B_i(¡$_i$) is the beliefs owned by player i = 1; 2: The right-hand side means that

each player i derives his decision s_i: Again, the symbol ` means that the entire sentence is provable.

Jan: Shinzuki, please stop for a moment and let me digest your current statement. First, I like the fact that it does not seem to involve common knowledge at all. Our previous discussions led me to …nd that common knowledge is only in the mind of a player. This suggests that we can do away with common knowledge operators and focus only on beliefs. This appears to be what you have done. However, you also replaced g^0 by $¡^0$, and that worries me. If you included some "common knowledge" there, then I would not be so happy.

Shinzuki: We can suppose that $¡^0$ contains purely objective statements and no instances of "common knowledge" or beliefs.

Jan: Good. Then I can go further and try to understand (4) now. Let me put (1), (2), (3), and (4) next to each other on the blackboard.

(1) ` $g^0;B_1C(g^1);B_2C(g^2)$! $C(Nash(s_{12};s_{22}))$:

(2) ` $g^0;B_1C(g^1);B_2C(g^2)$! $B_1C(Nash(s_{12};s_{22}))\wedge B_2C(Nash(s_{12};s_{22}))$:

(3) ` $g^0;B_1(g_1^2);B_1C(g_1^2;g_2^1)$! $B_1(Dom_1(s_{12}))\wedge B_2C(Nash(s_{12};s_{22}))$:

(4) ` $¡^0;B_1(¡_1);B_1(¡_2)$! $B_1(D_1(s_1))\wedge B_2(D_2(s_2))$:

Mathematically speaking, both (2) and (3) appear to be special cases of (4), but (1) is not.

Shinzuki: Uhum.., (1) is not a special case. However, we can include (1) by changing (4) slightly.

Jan: Ha, ha, ha. I do not care about generality. Rather, I am curious about the treatment of strategies here. I …nd that in (4) each player i has a decision s_i, while in (1) and (2) each player thinks about the Nash strategy pair $(s_{12};s_{22})$; and in (3), player 1 thinks about s_{12}, but 2 thinks about $(s_{12};s_{22})$: In this sense, they are asymmetric. More speci…cally, in (3) s_{12} is 1's decision, but what is $(s_{12};s_{22})$ for player 2? I ask this because player 2 cannot choose s_{12}:

Shinzuki: That is a good question, since I know the answer. The strategy s_{12} appears as a prediction of 1's decision making in 2's mind. Player 2 doesn't choose s_{12}; but he thinks about 1's decision making since he requires it to predict 1's decision making. To a large extent, this argument is rather standard from the game theoretical analysis of Nash [12].

Jan: But if (4) is a generalization of (2) and (3), then why doesn't it include predictions?

Shinzuki: Aha, (4) may include predictions in $D_i(s_i)$; though they are not explicitly written. In the case of (3a), since a dominant strategy is the decision criterion, the prediction part is not included at all.

Jan: OK, so again we move further from common knowledge. Do we really need common knowledge?

[Shinzuki speaks now in an authoritative tone]

Shinzuki: Your question is naive. The issue has many aspects. First, we need to think about the treatment of common knowledge and common beliefs, and more fundamentally, the di¤erence between knowledge and belief. Second, we should think about the relationship between common knowledge and decision making. We found by our analysis of (3) where player 1 has a dominant strategy, that common knowledge is "not necessarily" needed for decision making. Are you interested to …nd when common knowledge is necessary for decision making?

Jan: I would like to hear about the distinction between knowledge and belief now, but OK since you seem to want to, why don't you tell me about when common knowledge is needed?

Shinzuki: [Spoken with a German accent] Yah, yah..., I promise to discuss the distinction between knowledge and belief later. [In normal voice] But for now, let's discuss when common knowledge is needed. You will …nd that the argument is amazingly standard in the game theory literature. Consider the following form of prediction-decision making:

(a) 1 maximizes his payo¤ predicting that 2's decision is governed by (b);

(b) 2 maximizes his payo¤ predicting that 1's decision is governed by (a).

Jan: Hold on..., (a) appears in (b), and (b) appears in (a). This seems to involve circular reasoning. The sentence (a) can be plugged into (b), and (b) into (a), and so on, forever. This process yields an in…nite regress, and I have a great head ache with such circularities and in…nite regress. Is there any meaningful solution?

Shinzuki: Your concern is valid. To treat that in…nite regress, we need a common knowledge extension of an epistemic logic. In such an extension, we …nd a complete solution which turns out to be the common knowledge of Nash equilibrium. If (a) and (b) are restricted to occur in the mind of a single player, then the solution is the personal belief of the common knowledge of Nash equilibrium, i.e., $B_iC(\mathrm{Nash}(s_1; s_2))$.[6]

[In an authoritative tone]

Exactly speaking, (a) and (b) are properties required for a type of decision criterion. It is my claim that if such a decision criterion is adopted, then common knowledge is necessarily involved.

[Shinzuki starts speaking in a low voice with a critical tone]

Some people argued that common knowledge is unnecessary for Nash equilibrium. But that is not the issue! The issue is the necessity of common knowledge

[6] See Kaneko [5].

for decision making, not Nash equilibrium. Nash equilibrium is Nash equilibrium, as an apple is an apple in the analogy of apples. Nash equilibrium should be distinguished from the description of decision making.

Jan: You seem to mean that the necessity of common knowledge may depend upon the decision criterion used. Hummm,....the dominant strategy criterion does not involve common knowledge. However, it might not enable a player to make a decision in a game. Is there any decision criterion that always gives a decision and does not involve common knowledge?

Shinzuki: Actually, there is the simplest example called the default decision criterion. It recommends a pre-speci…ed strategy, say, the …rst strategy, without requiring the decision maker to think about anything. If you think it is simple, I can give more examples based on this.

Jan: I am getting tired...., but the last point sounds very nice. Maybe we should have a co¤ee break. But before that, I have one thought I would like to bring out. Common knowledge involves an in…nite regress, or in…nite depths of nested beliefs structures. On the other hand, if the dominant strategy decision criterion or the default criterion is adopted, then no common knowledge is required, since there is no nesting of beliefs, or the depth of beliefs is only 1.

Shinzuki: Yes, you are correct. Would you like to see an example involving a …nite depth of beliefs greater than 1 without using the default criterion?

Jan: I am tempted by co¤ee, but please go on.

Shinzuki: The example is very simple and obtained from changing only one payo¤ of player 2 in the prisoner's dilemma:

	s_{21}	s_{22}
s_{11}	(5;5)	(1;$\underline{2}$)
s_{12}	(6;1)	(3;3)¤

Figure 0^0: $g^{0^0} = (g_1^{0^0}; g_2^{0^0})$

In this game, player 2 has no dominant strategy, but if player 2 believes that player 1 follows the dominant strategy criterion, then 2 predicts s_{12} as 1's decision, and based on this prediction, he chooses s_{22}: This is written as

(5b) ` $B_2(g_2^{0^0}[B_1(g_1^{0^0}))$! $B_2(B_1(Dom_1(s_{12}))\wedge Best_2(s_{22} \mid s_{12}))$:

I did not write 1's statement because it is too long.

Jan: OK, $B_1(Dom_1(s_{12}))$ in the scope of B_2 is the prediction of player 2 about 1's decision making. The depth of nested beliefs is 2 here, isn't it? Then, $Best_2(s_{22} \mid s_{12})$ means s_{22} is the best strategy to the prediction s_{12}. It all makes sense. I think, I can now …nd more examples which require …nite depths of nested belief structures. This appears to capture some aspect of bounded rationality, doesn't it?

Shinzuki: Yes, it is one aspect of bounded rationality. You will …nd more examples in [7].

Jan: Now I remember how tired I am. I want to ask one more small question before co¤ee, since I feel we have reached some deep implications for game theory. My last question is about the multiplicity of... uhum..., what?

Shinzuki: You are really tired. I think you want to say the multiplicity of candidates for decisions.

Jan: That's right, in fact your wording is better than what I was thinking. Suppose we have a game like the battle of the sexes with two Nash equilibria:

	s_{21}	s_{22}
s_{11}	$(2;1)^{¤}$	$(0;0)$
s_{12}	$(0;0)$	$(1;2)^{¤}$

Figure 4: $g^4 = (g^4_1; g^4_2)$

Even if each player has a belief of common knowledge of a Nash equilibrium, each may have a di¤erent Nash equilibrium in mind. Then it might seem that each can make a decision to use his equilibrium strategy. But the combination is not a Nash equilibrium. How do you treat such a situation?

Shinzuki: You need an additional assumption such as the common knowledge of one Nash equilibrium, for example, $(s_{11}; s_{21})$. This may be included in ¡¡ of (4). If both players have the same common knowledge assumption, then the resulting outcome is $(s_{11}; s_{21})$.[7] However, if only player 1 has this, the outcome $(s_{11}; s_{21})$ may not be reached, yet player 1 chooses s_{11} and predicts s_{12} to be chosen by player 2. To obtain such common beliefs in one strategy combination needs communication, and such communication is external to epistemic logic.

Jan: Let's go for co¤ee now, to avoid more divergence. Do you know any good places?

Shinzuki: That is a very di¢cult question. Anyway, we can …nd co¤ee in the village.

Act 4: General Principles

[Shinzuki and Jan are walking back from a co¤ee shop]

Jan: It was good co¤ee, but the co¤ee shop was too crowded. I have an interesting idea about how to control the number of customers. Can you imagine it?

Shinzuki: Yes, I can, of course. It is that the price should be increased according to the economic principle that when price rises, demand will fall.

[7] See Kaneko [4] for the treatment of games with uninterchangeable Nash equilibria.

Jan: That is one way. Another way is to decrease the quality of co¤ee. Then some customers would leave.

Shinzuki: Ha ha, who makes such a stupid decision?

Jan: Yes, yes, a co¤ee shop in my town did this really. It succeeded in decreasing the number of customers. I myself went there only once after the reduction in quality. Can you believe this control?

Shinzuki: It is di¢cult to believe. Now I should impose a constraint on you not to digress from our main problem.

Jan: Sorry. In fact, this is my revenge for your past digressions.

Shinzuki: Indeed, it is a nice lesson in understanding the transgression of digression. I will think about how to get you back.

Jan: Now, let me try to recall what should be discussed. Uhum..., you promised to discuss the distinction between knowledge and belief.

Shinzuki: Good, I remember our problem, too. In an European tradition of philosophy it is standard to de…ne

(6): knowledge is a justi…able true belief.[8]

In this de…nition, belief is more basic than knowledge. For the moment please take a "belief" as something held in the mind of the believer. I want to talk …rst about the word "true". This notion of truth is taken from the perspective of the objective observer, or at least some thinker other than the one holding the belief.

Jan: OK, you are saying that the truth in (6) is external to the believer. There might also be a truth included within the belief of a person. But this truth is beyond dispute for him and we cannot discuss the truth or falsity of such beliefs within his perspective. We can only objectively distinguish between false and true beliefs if we take the word "true" written in (6) to be an external one. A true belief in this sense is a candidate for knowledge, isn't it? I think this is compatible with what we discussed before.

[Jan and Shinzuki arrive at the laboratory]

Shinzuki: Very good. In my previous statements, such as (2) and (3), there are at least three references to truth. One is from the perspective of the outside objective observer, and then there is one from the perspective of each player. In (2), g^0 is the situation true for the outside observer. From this perspective, the content, g^i; of $B_iC(g^i)$ is false, but, from the perspective of each player i his belief $B_iC(g^i)$ is true and beyond dispute. We discussed the falsities involved in (3) already.

[8] It is more standard to de…ne "knowledge" to be a "justi…ed" true belief, though the de…nition has been debated for a long time (see papers in Moser [10]). We adopt the word "justi…able" rather than "justi…ed", since whether or not a belief is judged to be "knowledge" is determined by the investigator. As it is discussed below that "truth" is stated externally, it is more natural to view a "justi…cation" as an external concept also.

Jan: It might help my understanding to consider the case where all beliefs are true even from the perspective of the outside observer. Consider the following:

(7): ` $g^0;B_1C(g^0);B_2C(g^0)$! ??

I put ?? in the right-hand side since the conclusion part is irrelevant to the point I want to make now. According to your explanation, the assumption part of (7) is indistinguishable from the common knowledge of g^0. Is there any meaningful di¤erence between this and simply writing $C(g^0)$?

Shinzuki: Actually, it can be proved that the set $g^0;B_1C(g^0);B_2C(g^0)$ is equivalent to $C(g^0)$; where the notation $g^0;B_1C(g^0);B_2C(g^0)$ is an abbreviation of g^0 [$B_1C(g^0)$ [$B_2C(g^0)$: Moreover, this is equivalent to $CC(g^0)$; and to $CC...C(g^0)$ with any …nite number of applications of C.[9]

Jan: Really? It is interesting. In this case, everything is OK with me and it also seems compatible with the standard "implicit" game theoretical arguments. With this new understanding, I would like to consider more cases of false beliefs. Let's return to the assumption part, $g^0;B_1C(g^1);B_2C(g^2)$, of (2). Because there are false beliefs involved, it seems possible that one of the players might notice this inconsistency and the assumption set would fall apart. How do you guarantee that the assumption set is all compatible and might persist?

Shinzuki: I am glad you are now getting through the gate of epistemic logic.

Jan: You are ‡attering me, but I am still pleased with my …nding, and perhaps too at your noticing my …nding. Anyway, please continue.

Shinzuki: OK. The assumption set $g^0;g^1;g^2$ is inconsistent, and so is the set $g^0;C(g^1);C(g^2)$:[10] If we start with either of these assumption sets, the theory is nonsense since anything, including absurdities, can be derived from a set of inconsistent assumptions. To prevent such absurdities, and allow the persistence of a set of assumptions, it su¢ces to prove that the set is consistent. If consistency is shown, then no player will ever notice an inconsistency and all will be …ne. This is really the starting point of the research in the …elds of "epistemic logic and game theory". We use some developments from proof theory and model theory to guarantee the consistency of $g^0;B_1C(g^1);B_2C(g^2)$ which can be found in [5].

One notable …nding is that to obtain the consistency of this set, we need to drop one famous axiom from our epistemic logic:

(8): $B_i(A)$! A.

[9] There are two approaches to extending epistemic logic with common knowledge: the …xed-point approach and the in…nitary approach. For the former, see Fagin et al [2], Meyer-van der Hoek [11] and [9]. For the latter as well as the relationship to the former, see Kaneko et al [8].

[10] Throughout this paper, we assume that C(A) means the common knowledge of A rather than the common belief of A: In our context, the common belief of A is de…nable as $B_1C(A)$^$B_1C(A)$: Conversely, if we start with the common belief of A as a primitive, then the common knowledge of A is de…nable as $C_B(A)$ ^ A: See Kaneko et al [8] as well as Section 5 of Kaneko [5].

This is often called the truth axiom or veridicality axiom. If (8) is assumed, every belief is assumed to be true in the eyes of the outside observer, and false beliefs cannot be discussed in the epistemic logic.

Jan: Sorry to interrupt you, but another point which I want to mention now is about the meaning or interpretation of the consistency of an assumption set, and the viewpoint from which it is derived.

I can now think of four meaningful di¤erent perspectives and assumption sets. The assumption set for player 1 is clearly $B_1C(g^1)$, and for player 2 it is $B_2C(g^2)$. The last two perspectives involve the outside observer who may have either g^0 alone or the entire set $g^0;B_1C(g^1);B_2C(g^2)$. While we can imagine an outside observer who knows only the true game g^0, we can also imagine an objective observer who knows all of $g^0;B_1C(g^1);B_2C(g^2)$. This second type of objective observer is rather close to the position I am taking in my analysis, and the one you seem to be taking in your analysis.

Shinzuki: Your last distinction is very nice. The consistency of $g^0;B_1C(g^1);B_2C(g^2)$ is proved from the viewpoint of the second objective observer you mentioned, even though I may have seemed to take the perspective of the …rst observer.

[Jan is pleased]

Jan: I am slightly curious about how to prove the consistency of $g^0;B_1C(g^1);B_2C(g^2)$, and perhaps this will help me understand what I just tried to describe: Can you give a brief explanation of the method of proof?

Shinzuki: Yes, I can. A proof needs a lot of details. However, believing the consistency is much like believing the logical possibility of the Konnyaku Mondô. The Konnyaku Mondô involves a set of false beliefs that is just like the assumption set $g^0;B_1C(g^1);B_2C(g^2)$: By admitting the logical possibility of the Konnyaku Mondô, you also admit a belief in the possibility of its consistency. When you heard the story, you most likely searched for some way to make sense of the story. This is what a proof of consistency does. The situation arrived at for making sense of the story, is called a model. If we can construct a model of $g^0;B_1C(g^1);B_2C(g^2)$; then we can use the soundness theorem to obtain the consistency of $g^0;B_1C(g^1);B_2C(g^2)$:

Jan: Stop, stop. It seems a lot or things are needed for that step. Please postpone your explanation of the construction of a model and the soundness theorem to sometime in the future. I am learning a lot now from your description and by thinking about the story in this new light. Can we change our direction slightly and discuss the word "justi…able" in (6).

[Shinzuki shows his disappointment by raising his hands to his head]

Shinzuki: Arghhh..., I am slightly disappointed, since that construction is my favorite part. Perhaps later you will appreciate it more. OK, let's consider "justi…able" in (6). It is not very exciting, technically speaking. Usually, "justi…-able" is quite ambiguous, but it is clear-cut in the epistemic logic approach since that approach unearths everything. By a justi…able belief, say A; for player i,

we mean that player i has some argument, or justi…cation for A from his basic beliefs. The basic beliefs are written in ¡¡ of (4). For example, if player i has a mathematical proof for A from ¡¡; then this A is a justi…able belief.

Jan: It is clear-cut. A belief is justi…ed when, for example, a player has a proof of it, uhum..., this proof is taken from his basic beliefs, right? So mightn't we ask for a justi…cation for a basic belief?

Shinzuki: No, basic beliefs are not justi…ed in this sense.

Jan: But you claimed that the epistemic logic unearths everything.

Shinzuki: Sorry. I should say it unearths everything that can be unearthed.

[A bit surprised by Shinzuki's statement, Jan retorts]

Jan: Ha ha, that sounds like a tautology. Is it everything or nothing? It seems to me that it refuses to dig.

Shinzuki: You are right. I retreat by saying epistemic logic unearths everything before the consideration of basic beliefs ¡¡. We need to look for some other sources for basic beliefs.

Jan: What other sources for basic beliefs do you have in mind?

Shinzuki: This part is not well developed, though it has been discussed since the age of Plato. One source is one's experiences. This process is often called "induction" and it means to obtain a general law from …nite experiences. Please note that this "induction" di¤ers from the mathematical induction principle. While the latter principle has the same name, it is essentially a principle of deduction. In fact, the latter may be included in ¡¡;[11] but the former is external to the logical framework and is the process of constructing and revising the basic beliefs in ¡¡.[12]

Jan: I am surprised to hear that the mathematical induction principle is one of deduction. The other "induction" you speak of is not deductive at all. When I learned the distinction between induction and deduction at university, both were described as processes of scienti…c reasoning.

Shinzuki: One di¤erence here from what you learned is that our target is a human inductive process rather than a scientist's inductive one. The latter requires careful statistical treatments, but the former is rather a bold process since a great generalization is often made from a few experiences. We all have this tendency to generalize. For example, you went to the co¤ee shop only once after it decreased the co¤ee quality, but the shop might have increased the quality later. Are you sure that the co¤ee quality there is still bad?

Jan: Touche! I understand your distinction between human and scienti…c inductive processes.

[11] To include the mathematical induction principle in ¡¡, we need predicate epistemic logics. For such predicate epistemic logics including common knowledge, see Kaneko et al [8].

[12] The revision of beliefs has been discussed in the literature on belief revision. See Schulte [15] for a good survey.

Shinzuki: I would like to give one more example in this line of reasoning to deepen your understanding. Suppose that you meet two people of an ethnic group that is new to you. You observe di¤erences in their appearances as well as their behavior relative to the standard in your community. You might conclude that all people of that ethnicity have similar appearances and behavior.

Jan: I understand that these generalizations are rampant in our lives, but on the positive side, they allow us to expand and revise our basic belief set ¡¡.

Shinzuki: Your point is taken, but I would rather like to point out that once again we have arrived at some false beliefs. The generalities obtained by induction involve false beliefs in the sense that they are not valid from an objective viewpoint. These false beliefs are related to sociological problems of discrimination and prejudice. The inductive method I have been speaking of is formulated in [6], which the authors call the inductive game theory. In that paper, the emergence of racial prejudices is discussed, but connections to the epistemic logic are not yet fully reached.

Jan: It sounds interesting. I will look at the paper. However, I doubt I can attribute all my beliefs to my past experiences. My experiences are very limited, yet I seem to have a limitless fountain of beliefs. I even have some on things I never experienced, like what it would be like to ‡y.

Shinzuki: Yes, experiences are limited for an individual being, but we also have communication and education. The experiences of former generations have been communicated to later generations through discussions, books, and other sources. A lot of "trial and error" has been taken by our ancestors, and many of these experiences have been passed on. For example, our distinction between poisonous and edible mushrooms is possible only after many trials and some fatal errors by previous generations.

Jan: The accumulation of such experiences are taught to us. Great! We have been making progress by our experiences.

Shinzuki: You are right to the extent that what we have discussed is quite a standard view of progression. However, I want to point out that communication and education are powerful, but have their own limitations. For communication and learning as well as for one's own memory of experiences, language is a useful and a necessary tool. Language consists of primitive symbols and grammars, just as a logical system consists of symbolic expressions and inference rules. In this sense, a logical system can be regarded as the idealization of a language.

[Having tuned out slightly, Jan notices Shinzuki is staring at him waiting for a reply]

Jan: What is your point?

Shinzuki: My point is as follows. Suppose somebody has experiences. However, he translates his experiences into language in order to communicate those experiences to others or even just to memorize them for himself. In this translation, something is lost as well as gained. Something is lost by the fact that the

communicator must choose some part of his experience in order to be understood by others. Something is gained by adding the structure required to express the experience in a language understandable by others. Raw experiences can never be transferred to other people.

Jan: It would be nice if you could relate it directly to what we have been discussing.

Shinzuki: OK, I will continue. People having experiences construct a simple story explaining experiences, and other people are taught this story through communication or education. This is a source of the basic beliefs ¡¡ of (4).

[Shinzuki's voice becomes louder]

Now, I stress the story is coherent, but it must also be simple so it can be memorized or communicated. This implies that ¡¡ is innately subject to some falsities. If the story is extremely accurate and has every element of experiences, it is useless since it is overwhelmingly long. Also, everybody has a conscious or unconscious tendency to adjust the story so it is more comfortable for him or her. Such adjustments were described in a funny, but sad, manner in Akira Kurosawa's movie "Rashoumon".[13]

[Shinzuki is roaring with enthusiasm, and Jan feels slightly taken aback]

Jan: I understand what you want to say. My view of progress may be too naive. Does your argument have any relationship to the Konnyaku Mondô?

Shinzuki: Yes, it does! You are ...nally becoming very sensitive to the contradictory elements in our discussions. In the Konnyaku Mondô, the exchange of gestures is a communication. It represents the symbolic nature of language. Each person constructs a story from the gestures, which includes falsities. This example is an extreme one, but it shows that communication is never entirely free from fallacious elements.

Jan: Yes, indeed, but still a lot can be communicated by discussions like the ones we have been having.

Shinzuki: I am afraid we might be having nothing more than a Konnyaku Mondô. Just like you, I believe we have had meaningful discussions, but in actuality, the whole time I might have been talking about Konnyaku products in epistemic logic terms..., and you, my friend, might have been discussing the battle of the sexes in game theoretical terms.

Jan: You are too cynical! I think we should stop for today. Why don't we go to the public bath tonight? The public bath increases the quality of our life here very much. I hope the town never decides to decrease the quality of the water to control crowding problems.

Shinzuki: Ha, ha, it is a great idea to meet at the public bath. It helps you understand the Japanese culture, doesn't it? Shall we meet there around 8pm?

[13] The original short story for the movie "Rashoumon" is called "Yabunonaka" by Ryunosuke Akutagawa. He wrote a di¤erent story with the title "Rashoumon".

References

[1] Chellas, B., (1980), Modal Logic, Cambridge University Press, Cambridge.

[2] Fagin, R., J.Y. Halpern, Y. Moses and M. Y. Verdi, (1995), Reasoning about Knowledge, The MIT Press, Cambridge.

[3] Hughes, G. E. and Cresswell, M.J., A Companion to modal logic. London: Methuen &Co. (1984).

[4] Kaneko, M., (1999), Epistemic considerations of decision making in games. Mathematical Social Sciences 38 (1999), 105–137.

[5] Kaneko, M., (2002), Epistemic Logics and Game Theory: Introduction, Economic Theory 9, 7-62.

[6] Kaneko, M., and A. Matsui, (1999), Inductive Game Theory: Discrimination and Prejudices, Journal of Public Economic Theory 1, 101–137.

[7] Kaneko, M., and N-Y. Suzuki, (2002), Bounded Interpersonal Inferences and Decision Making, Economic Theory 9, 63-104.

[8] Kaneko, M., T. Nagashima, N.-Y. Suzuki and Y. Tanaka, (2001), A Map of Common Knowledge Logics. To appear in Sudia Logica.

[9] Lismont, L. and Mongin, P.,(1994), On the logic of common belief and common Knowledge. Theory and Decision 37, 75-106.

[10] Moser, P. K., (1986), ed., Empirical Knowledge: Readings in Contemporary Epistemology. Rowman & Little...eld Publishers, Inc. Chicago.

[11] Meyer, J.-J. Ch. and van der Hoek, W., Epistemic logic for AI and computer science. Cambridge: Cambridge University Press (1995).

[12] Nash, J. F., (1951), Noncooperative Games, Annals of Mathematics 54, 286–295.

[13] Plato, (1941), The Republic of Plato, Translated by F. M Cornford. London: Oxford University Press.

[14] Savage, L., (1954), Foundations for Statistics, Dover Publication Inc. New York.

[15] Schulte, O., (2002), Minimal Theory Change and the Pareto Principle, Economic Theory 9, 105-144.

[16] Terutoshi, Y. Okitsu K. and Enomoto, S., (eds.), Collections of meiji-taisho comic stories (in Japanese). Vol. 3. Tokyo: Kodansha (1980)