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Perfect equilibrium has a lexicographic preference for shorter paths of play

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Abstract

A simple example is used to illustrate one clear distinction between perfect equilibrium and sequential equilibrium. Perfect equilibrium has a lexicographic preference for shorter paths of play. We use this example to disprove the conjecture that: in a game of perfect information, every subgame perfect equilibrium is a perfect equilibrium.

By now it is well known that perfect equilibrium and sequential equilibrium are almost equivalent concepts (Kreps and Wilson (1982)). This is not surprising since both are based on the concept of a perturbed game and derived in almost the same fashion. We illustrate, by a simple example, a clear and simple manifestation of the di¤erences between the two equilibrium concepts. By this example, we disprove the conjecture that: in a game of perfect information, every subgame perfect equilibrium is a perfect equilibrium. Our observation is in the same spirit as Mertons' (1995) observation that a player may use a weakly dominated strategy in a perfect equilibrium.

Perfect equilibrium, unlike sequential equilibrium, turns out to have a lexicographic preference for equilibria with shorter paths of play. The key reason for choosing the shorter path of play in a perfect equilibrium is the fear of multiple mistakes. A player looking for a perfect equilibrium, must optimize in the environment of slight mistakes and thus is driven to a slight degree by the fear of multiple mistakes. Consider the following conjecture:

Conjecture 1 *In a game of perfect information, every subgame perfect equilibrium is a perfect equilibrium*

We use the following decision problem in Figure 1 to illustrate the falsity of this conjecture.

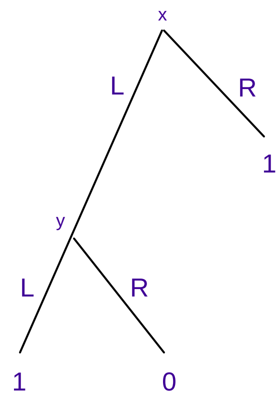

Figure 1

This is a one-player game with two subgame perfect equilibria: (L,L) and (R,L). Let's see that the strategy (L,L) is not a perfect equilibrium of this game. Consider any perturbed game where $\varepsilon_{xL}; \varepsilon_{xR}; \varepsilon_{yL}; \varepsilon_{yR}$ are the minimum choice probabilities of L at x, R at x, L at y, and R at y respectively. The expected utility of a strategy placing maximum probability on L at each information is:

$$(1 - \varepsilon_{xR})(1 - \varepsilon_{yR}) + \varepsilon_{xR} = 1 - \varepsilon_{yR}(1 - \varepsilon_{xR}): \qquad (1)$$

We show that the strategy of choosing R with maximum probability at x, and L with maximum probability at y gives a higher expected utility in any perturbed game with sufficiently small mistake probabilities. This strategy gives the expected utility:

$$(1 - \varepsilon_{xL}) + \varepsilon_{xL}(1 - \varepsilon_{yR}) = 1 - \varepsilon_{yR}\varepsilon_{xL}: \qquad (2)$$

Clearly, (2) dominates (1) in value when the mistake probabilities ε_{xL} and ε_{xR} are sufficiently small that $\varepsilon_{xL} < (1 - \varepsilon_{xR})$. Thus (L,L) cannot be a perfect equilibrium.

The reason that (L,L) fails to be a perfect equilibrium is that it requires two decisions to get to the goal payoff of 1, and thus a greater possibility for mistakes. The strategy (R,L) typically involves only one decision to get to the goal. It is only in the case that the player makes a mistake at his first decision point that a second decision must be made under (R,L).

Notice that the concept of sequential equilibrium does not rule out the (L,L) equilibrium. For a sequential equilibrium, mistakes are used only to determine beliefs at an information set. In a game of perfect information, any mistakes will lead to the same beliefs at any decision point. Optimality is only considered in the limit game which involves no mistakes and hence both (L,L) and (R,R) continue to be sequential equilibria. This proves the following.

Proposition 2 *In a game of perfect information, every subgame perfect equilibrium is a sequential equilibrium.*

We now show by the following slightly modified example, that the preference of perfect equilibrium for shorter paths of play only comes when two paths of play lead to the identical payoff. In this sense, it is a lexicographic ordering.

In this modified game, the endnode reached after y by a choice of L at y gives a slightly higher payoff of $(1+e)$ where $e > 0$. Clearly, in this game the unique subgame perfect equilibrium is (L,L). Since perfect equilibria exist in games of perfect information and they are a subset of the subgame perfect equilibria, it follows that this is also the unique perfect equilibrium. Hence, we find that a player does not trade off additional payoff against shorter paths of play in a perfect equilibrium. It is only in the case that the payoffs are identical that he will choose the shortest path.

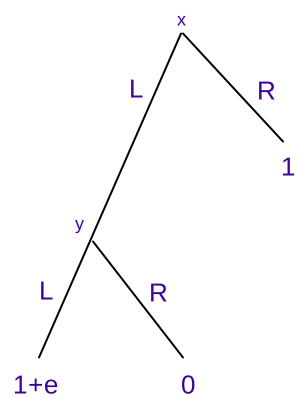

Figure 2

References

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[3] Selten, R. Reexamination of the perfectness concept for equilibrium points in extensive games, International Journal of Game Theory 4, 22-55 (1975).