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# The Boyle's Law Simulator: A Dynamic Interactive Visualization for Discovery Learning of Experimental Error Analysis

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## Abstract

As a follow-up to the classic Boyle's law experiment, students can investigate experimental error in the virtual realm of a spreadsheet. Students engage in numerous higher-order thinking and science process skills as they work through the simulation.

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## 1. Introduction

Boyle's law relates the pressure ( $P$ ) and volume ( $V$ ) of a trapped amount of gas (moles,  $n$ ) at constant temperature ( $T$  in Kelvin). It is classically demonstrated using a syringe connected to a pressure measuring device (gauge, probeware sensor) and discussed in beginning chemistry and physics courses [2, 3, 8]. From the mathematical point of view, Boyle's law illustrates a power law of the form  $y = ax^b$  where  $a$  depends on experimental conditions and  $b$  is  $-1$ . This is typically discovered by students from the graphical analysis of the collected data using  $PV = k$  or  $P = k/V$  where  $k$  is a constant. In the Boyle's Law Simulator, instructors can enhance the typical Boyle's law experience to have students understand why the trapped amount of gas and temperature must remain constant to see the  $PV$  relationship and determine how other experimental errors can influence the results. This article outlines the concept of constructing an interactive Excel spreadsheet and the type of questions to probe the error analysis.

## 2. Developing the Learning Tool

An interactive Excel spreadsheet or Excelet has been developed as a discovery learning tool to explore experimental error as a post-laboratory activity after performing the Boyle's law experiment. This Excelet is computationally based (no macros or programming) with a number of interactive features from the forms toolbar (usable on both PC and Mac platforms) [5]. A number of manipulable variables ( $n$ ,  $T$ ,  $V_{tubing}$ , and *noise factor*) have been generated that influence the calculation of the pressure, resulting graph, and any calculated results including the regression. The interactive features are shown by the grey boxes in Figure 1. It is recommended that the reader opens the accompanying Excelet and goes to the simulation I tab. The wherewithal for creating Excelets can be found at the Developer's Guide to Excelets website [6], which includes tutorials, illustrated

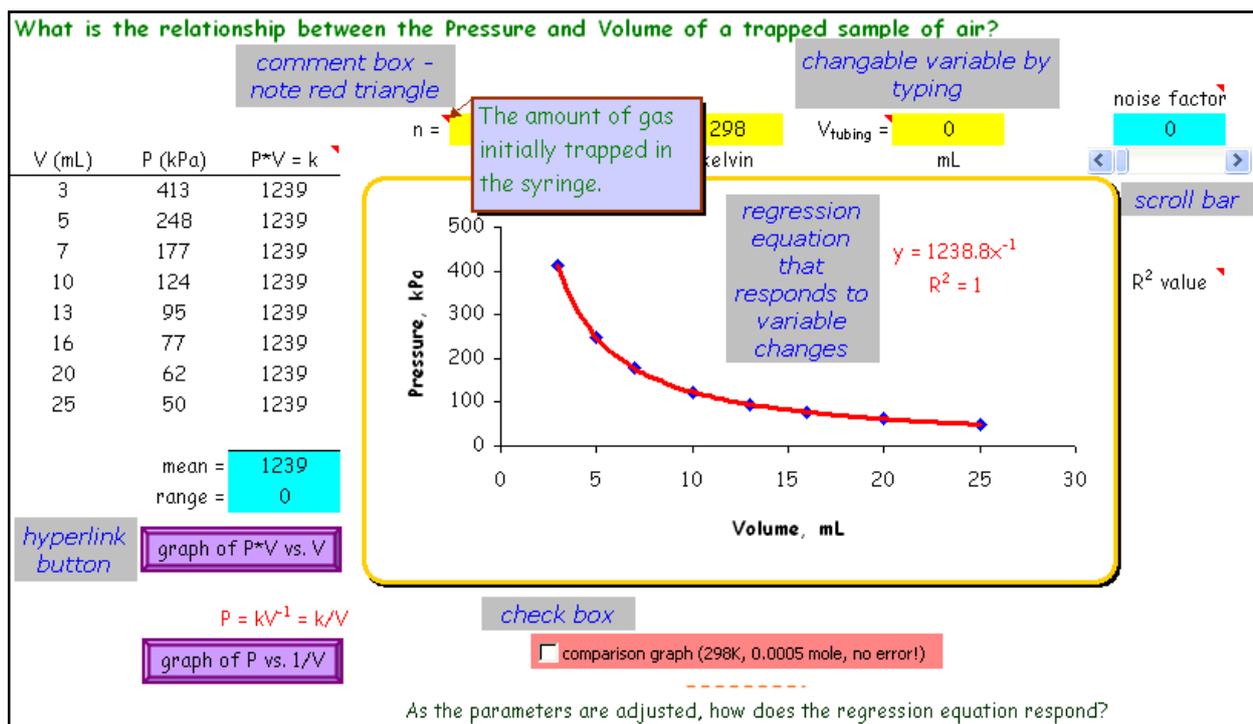


Figure 1 – Interactive Features on simulation I tab (highlighted in grey boxes)

instructions, and many more examples plus materials to get students into mathematical modeling of data. Students do not need a great deal of Excel experience to use Excelets, as a short introduction to the use of the interactive features and how to explore a variable can get them going.

### 3. Implementing the Discovery Learning Process

Discovery learning is accomplished by the mode of questioning that accompanies the Boyle’s Law Simulator. Through numerical experimentation using a manipulable variable, students can predict-test-analyze how this variable influences the data and graph or any calculated parameters. This is the interactive dynamic feature of Excelets.

## THE BOYLE'S LAW SIMULATOR

<b>Predict</b>	How will increasing (decreasing) the variable influence the data? Make your prediction and sketch a graph of it
<b>Test</b>	Increase (decrease) the value of the variable.
<b>Analyze</b>	What happened? Describe the change, if any, in sound mathematical language. How did the result compare to your prediction? This is the interpretation and analysis.

Here is a selection of possible questions (highlighted in blue italics) for students to ponder.

*Suppose the syringe leaks or the temperature changes? Predict what you think will happen if the syringe leaks and then decrease the amount of gas and see how the graph responds. Describe the change compared to the errorless condition. Now repeat for a temperature change.*

Within the Boyle's Law Simulator, there are four manipulable variables: the amount of trapped gas (in moles)

$n$  – the temperature (in Kelvin),

$T$  – the volume of the tubing,

$V_{tubing}$  and the *noise factor* (adjusted by moving the scroll bar).

It is assumed that students have a basic knowledge of elementary mathematical modeling of data (regression, goodness-of-fit) [5]. To assist students in their interpretation, a comparison graph with no error can be added to the graph via a check box. Remind students to explore a variable over a wide range of values. As the parameters are adjusted, how does the regression equation respond? Here students need to pay attention to the  $a$  and  $b$  values of  $y = ax^b$  and the value of r-squared as well, which addresses goodness-of-fit of the regression to the data. (Note: For the power regression, the value of r-squared is actually calculated from the ln-ln transformed data and linear regression. See the simulation I tab and scroll down.)

The ideal gas law is  $PV = nRT$  where  $P$  is the pressure in KiloPascals (kPa),  $V$  is the volume in liters,  $n$  is moles,  $T$  is the Kelvin temperature, and  $R$  is the gas constant (8.314 L-kPa-mol<sup>-1</sup>-K<sup>-1</sup>). Hence  $PV = k = nRT$  and a change in  $n$  or  $T$  will cause similar shifts in the curve and changes in the regression results. In these comparisons the r-squared value will not be influenced since the changes applies to all data collected. If these changes occurred during the experiment, r-squared would be influenced. Why? See the simulation II tab for a leaky syringe that starts to leak at a volume of less than 13mL.

Now as can be seen from the above analysis, these changes are experimental in nature and they influence the value of  $k$  in the results. Now let's examine true experimental error assuming no leakage and constant temperature during the experiment. We are reading the volume of gas trapped from the syringe; however,

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there is a small volume of gas in the tubing that connects the syringe to the pressure measuring device. This volume of tubing induces an error in the experiment. The error can be minimized by keeping the length of tubing as short as possible and it can be corrected by measuring it. The other problem is holding constant volume under manually generated pressure.

*How does the volume of tubing influence the graph and regression results? How does keeping the volume held at a constant value influence the results (at low volumes this is not easy is it!)? Is there any fluctuation in the syringe piston?*

These two experimental errors are common in student results, especially if students are not careful while making measurements. These two errors influence the  $a$  and  $b$  values of  $y = ax^b$  and the value of  $r$ -squared as well. The volume of tubing error is a systematic error which can be corrected for experimentally by measuring it. This is done by trapping gas in the tube at atmospheric pressure with the syringe set at zero. If you pull back on the syringe until the pressure is half its initial reading, the volume on the syringe is the volume of gas in the tubing ( $PV = k = P/2 * 2V$  and  $V_{tubing} = V_{syringe}$ ). If you keep the tubing short you can minimize this error and avoid the correction. The correction is a nuisance to novice learners. To correct the data the following must be done:  $V_{corrected} = V_{syringe} + V_{tubing}$  and then plot  $P$  vs  $V_{corrected}$  (see the volume of tubing tab for further information). Recently, Spencer has proposed a plot of  $PV$  vs  $P$ , which is very sensitive to error and the linear regression yields both the values of  $k$  and the  $V_{tubing}$  (shown on the simulation I and II tabs) [7].

*How do you know the volume of the tubing is a systematic error?*

Examine the  $k$  vs volume plot. Students would observe a consistent shift in the “ $k$ ” values from the errorless value of “ $k$ .” For more on types of errors, see Sinex [4].

*How do you know the fluctuation of the piston causes a random error?*

The fluctuation of the piston causes a random error in the data since the variation can be a smaller or larger volume. This added scatter in the data will naturally influence the  $r$ -squared value. The added random noise also influences the  $a$  and  $b$  values as well. The  $k$  vs volume plot shows a scatter of the  $k$  values around the errorless value.

Since results can yield a power that appears to be close to one (tubing volume error minimized), transform the data to a plot of  $P$  as a function of  $1/V$  and do a linear regression to derive the  $k$  from the slope of this plot. This can be compared to the mean  $k$  value from the data points.

## THE BOYLE'S LAW SIMULATOR

What determines the value of  $k$  in the Boyle's law experiment?

Since  $k = nRT$ , it depends on the laboratory temperature and the starting amount of air in the syringe (volume set on syringe at atmospheric pressure).

Students can now go back to their original experimental data and decide what possible errors could be influencing their results. This is a great mindset for students to develop. Students can make a comparison of "k" values from the various methods as well and compare to other student groups.

On the fit with Solver tab, students can analyze actual experimental data by fitting a rectangular hyperbola,  $y = k/x$ , to their data. The preset Solver adjusts the "k" to minimize the sum of the squared errors. A plot of the residuals (error) is also obtained and for a good fit should show a random pattern.

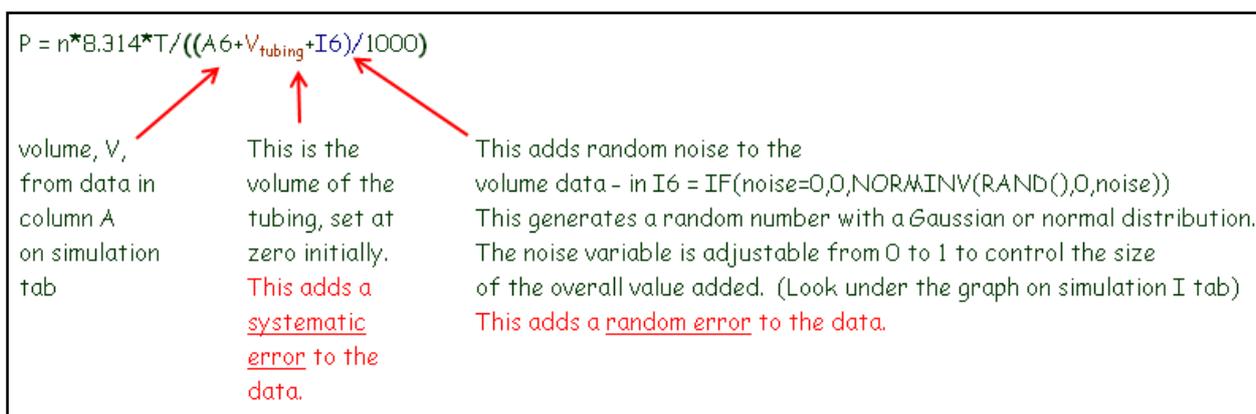


Figure 2 – The Addition of Error (on the simulation I tab)

#### 4. The Mechanics of the Simulation

The simulation uses this formula to calculate the pressure from the other variables:  $P = n * R * T / (V / 1000)$ . Since the value of the gas constant  $R = 8.314 \text{ L.kPa.mol}^{-1}.\text{K}^{-1}$ , we have to convert the volume,  $V$ , in mL to L by dividing by 1000. We can adjust  $n$  and  $T$  (the "held constant" variables) to see how the curve responds.

To get the error into the equation, we modify the calculation in the formula to include two additional terms as described in Figure 2. The use of the NORMINV(probability, mean, standard deviation) function with the RAND() function generates random numbers with a Gaussian or normal distribution [1]. With the mean set to zero, both positive and negative values are produced. The noise variable as the standard deviation then controls the size of the numbers generated.

For the leaky syringe on the simulation II tab, a factor ( $I6$ ) is added that varies with pressure when the leak starts as  $P = n * I6 * 8.314 * T / (A6 / 1000)$ . Look under the graph for details.

## 5. Some Final Thoughts

This computationally-based simulation of Boyle's law allows students to develop a deeper understanding of a simple experiment. Novice learners can start developing the mind-set to consider error in data and discover how errors influence data and the resulting model. It allows a multivariable approach, while examining the mathematics in a camouflaged manner. It supports the "Rule of Four" in mathematics, where the numerical, graphical, symbolic, and verbal aspects are all explored, all-the-while enriched by technology using a common spreadsheet application. The use of interactive Excel spreadsheets or Excelets [5, 6] creates an engaging pedagogy for learners. Instructors can generate their own questions and modify the interactive spreadsheet to meet needs of their students.

## 6. Acknowledgements

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